Zeno of Elea is most famous for his “paradoxes of motion,”¹ four arguments which purport to prove, contrary to sense experience, that motion across a continuum is impossible. In particular, one of Zeno’s arguments against motion can be singled out as exceptionally influential in the ancient world insofar as it was a midwife in Atomism’s birth. We learn from Aristotle that the argument “from Dichotomy”² played some role in the positing of atomic magnitudes (ατοµα µεγετηε) by the fifth century B.C. atomists,³ and the Dichotomy’s influence on Epicurean atomism is unambiguous.⁴ Thus, a precise understanding of Zeno’s Dichotomy, both its formulation and

1 It is we moderns who consider Zeno’s four arguments against the possibility of motion “paradoxes.” In antiquity, they were usually referred to simply as “arguments.” However, the term is not inappropriate because their conclusion that motion is impossible flies in the face of what seems to be and is believed to be the case.

2 For the name “Dichotomy,” see Phys. 239b18–20. It has also been called the “Stadium” paradox, see Topics 160b. Since Aristotle, our main source for Zeno’s paradoxes, refers to our paradox explicitly as the “Dichotomy,” I too shall do the same in what follows.

3 Cf. Phys. 187a1–3. There is a dispute about who Aristotle is really referring to in our passage, since he does not actually name the atomists or anyone else in particular. The Greek commentators believed that Aristotle was actually referring to Plato and Xenocrates (see Simplicius, in Phys. 133.30–142.27), while some modern scholars argue that the atomists are the ones in question (see Furley, Two Studies in the Greek Atomists, 81–2). No matter who is correct on this issue, it is not implausible to maintain that the fifth century atomists were at least motivated in some minimal sense by Zeno’s Dichotomy without making any pronouncements in answer to the question whether they postulated atomic magnitudes as a result of, or in answer to, the Dichotomy.

4 Cf. Letter to Herodotus 56ff. Of course, to say that Zeno’s Dichotomy influenced Epicurus is not to say that, for instance, Epicurus had a text of Zeno in front of him when he was thinking up his atomism.
assumptions, will serve us well historically by helping us to see more clearly an argument, or at least the spirit of an argument, that the Atomists had to face head-on. However, the Dichotomy is also an equally important piece of philosophy, and there is much to be gained philosophically by examining it in itself, divorced from the historical context in which it was framed. This paper will attempt to navigate between the Scylla of mere historical interest and the Charybdis of historically insensitive philosophical disputation with the hopes of accurately elucidating and persuasively dissolving Zeno’s Dichotomy. First I will discuss two possible interpretations of the paradox and argue that the second interpretation is more likely to be what Zeno intended. Having established the correct interpretation of the paradox, I will argue that the approach that most modern commentators take in trying to resolve the paradox is misguided and then present my own dissolution of the paradox.

**Preliminary Sketch of the Dichotomy**

The Dichotomy comes down to us from Aristotle in a few passages from the *Physics*:\(^5\)

...the first [argument] asserts that there is no motion owing to the fact that what moves must reach the halfway point before reaching the end.\(^6\)

...it is always necessary to traverse the halfway point, but these are infinite, and it is impossible to pass through an infinite number of points.\(^7\)

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\(^5\) See also *Topics* 160b and *Phys*. 233a 21–31. I shall save the latter passage for later discussion.

\(^6\) 239b11–13.

\(^7\) 263a5–6.
From Aristotle’s words we may construct the Dichotomy as follows. It is impossible to travel from the beginning $B$ of a track to the end $E$ of a track. For in order to reach $E$, one must reach the halfway point $H1$ between $B$ and $E$. But there is a halfway point $H2$ between $H1$ and $E$. Thus, one must also reach $H2$ before reaching $E$. But there is still another halfway point $H3$ between $H2$ and $E$, which the runner will have to reach in order to reach $E$, and so on, *ad infinitum*. In other words, there is an infinite sequence of points on the track, all of which the runner must cross, if he is to reach the end. But, it is impossible to cross all of the points in an infinite sequence of points. Thus, it is impossible to move from $B$ to $E$.

The Two Readings

One must first ask the question: What is it that Zeno claims to be impossible? Why is it impossible to complete the performance of crossing all points in an infinite sequence of points? There are two possible readings of the Dichotomy depending upon whether Zeno’s answer to this question primarily concerns the length of the track or the runner. That is to say, either Zeno is primarily making a claim about the track (and secondarily about the runner), i.e., that the track is infinitely long and for this reason a runner is unable to cross it, or he is primarily saying that a runner cannot cross the track because motion from $B$ to $E$ involves completing an infinite number of tasks one after another, which is presumably impossible. This distinction is important because each of the two readings involves different assumptions and must be handled differently. Let

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8 I shall presume that a progressive infinite sequence of points is intended (in which the dissection of the track proceeds from $H1$ to $B$), though the Dichotomy has also been read as an infinite regressive sequence of points (in which the dissection of the track proceeds from $H1$ to $B$)—cf. Sextus, *Pyrrh. Hyp* 3, 76 and Adv. Math. 10, 139–41. While it is not crystal clear which kind of sequence Zeno intended, Gregory Vlastos has presented a cogent argument for thinking that an infinite progressive sequence is the correct reading of the Dichotomy, cf. “Zeno’s Race Course” 201ff. in Allen and Furley vol. 2.
us initially discuss the first possible reading of the Dichotomy.

THE FIRST READING

According to the first interpretation of the Dichotomy, Zeno capitalizes on the same assumption that he used in one part of his argument of “large and small,” which we find in Simplicius:9

Unlimitedness in magnitude he proved earlier by the same method of argument. For having first proved that if what is had no magnitude, it would not even exist, he goes on: “But if it is, it is necessary for each to have some magnitude and thickness, and for the one part of it to be away from the other. And the same argument holds about the part out in front; for that too will have magnitude and a part of it will be out in front. Indeed it is the same thing to say this once and to go on saying it always; for no such part of it will be last, nor will there not be one part related to another. Thus if there are many things, it is necessary that they are both small and large; so small as not to have magnitude, so large as to be unlimited.”10

Here, Zeno attempts to show that the members of a plurality must be infinitely large. He assumes that if there are many things, each must have a certain magnitude (for he has presumably already shown that something with no magnitude cannot exist), and in this passage he argues that any magnitude has an infinite number of parts. In order for him to conclude validly that if something has magnitude then it is infinitely large, he must also be assuming that the sum of an infinite sequence of parts with magnitude is itself infinite.

9 In the argument of “large and small” Zeno attempts to prove that those who maintain the existence of many things are equally wedded to the contradictory beliefs that that they are both infinitely large and so small as to have no magnitude. We only discuss the arm of the paradox in which Zeno attempts to prove that the members of a plurality are infinitely large. For a more complete discussion of the argument of “large and small” see Kirk 266–69.

10 In Phys. 140, 34 = KRS 316, trans. KRS.
This is the relevant assumption with regards to the first reading of the Dichotomy. Accordingly, Zeno’s argument would have been that since the track is infinitely divisible in length, and because the sum of an infinite series of lengths with a certain magnitude is infinitely great, the length of the track will be infinitely great, i.e., the track will be infinitely long. Theoretically, because the track is infinitely long, no one will be able reach E. Thus, motion from B to E will be impossible.

Assuming that this is the correct reading of the Dichotomy, one may reply as follows. Only in some cases is the sum of an infinite series infinite, viz., when there is a smallest term. But in the case of the argument of large and small, and also in the Dichotomy, there is no smallest term; thus the series of magnitudes will add up to a finite number, i.e., the size of the individual or the length of the track.

However, such a reply is not as uncontroversial as it may seem at first. For saying that the sum of certain infinite sequences e.g., ½, ¼, ⅛, ⅛, etc., will equal a finite number is an imprecise way of stating the matter. What we actually have in those cases is a limit, which the partial sums of the terms of the sequence approach. Thus, the sequence of partial sums of the infinite sequence ½, ¼, ⅛, ⅛, etc., will approach 1; but this does not mean that the sequence has a last member or that the sum of the terms will ever actually reach 1.11

Fortunately, a less controversial reply is at hand. It is clear that the Dichotomy assumes the continuity of magnitude, because potentially infinite divisibility implies a continuity of magnitude. Thus, when we assume the continuity of magnitude, the fact that any given magnitude can potentially be divided into an infinite number of parts is trivial. But this fact does not permit us to infer anything about the actual length of a certain physical magnitude—especially extrapolating an infinite length—contra Zeno. Zeno’s Dichotomy, on this reading, gains

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11 See McKirahan in Long 147.
its plausibility from the tendency, ubiquitous in ancient thought, to blur the distinction between mathematical and physical entities and to infer facts about physical entities from facts about mathematical entities. However, one cannot infer that the length of the track will be infinitely long from the fact that any given continuous magnitude is infinitely divisible. Let us suppose we have two tracks, each of different lengths: one ¾ mile and another 1 mile. If we were to attempt to measure those two tracks in terms of mathematical points, all things considered, our measurements would be the same (i.e., both would equal c), since in a system of geometry, which assumes the continuity of magnitude, the cardinal number of points on two finite lines of differing lengths is the same. Using this line of reasoning, we would have to conclude that they are of equal length, yet they are not. Thus, if we were to use this information to make an inference about the determinate lengths of the physical tracks, we would be mistaken.

THE SECOND READING

Although the possibility that Zeno presented the Dichotomy along the lines of the first reading cannot be completely ruled out, because the Dichotomy does not come down to us in Zeno’s own words, it is more plausible that he focused primarily on the runner’s alleged inability to cross from one end of a track to the other. Zeno’s claim that a runner is unable to cross the track because motion from B to E is equivalent to successively performing an infinite number of individual tasks is suggested by two passages.

The first passage that explicitly attributes the second interpretation of the Dichotomy to Zeno appears at the beginning of the section in the Physics in which Aristotle offers his first response to the Dichotomy:

12 For a brief explanation of the “continuum” or “c,” see Salmon 260ff.
13 For the proof of this see Salmon 260–62.
Hence also Zeno’s argument makes a false assumption in asserting that it is impossible to traverse \(\delta\iota\epsilon\lambda\tau\epsilon\iota\nu\) infinite points or to touch infinite points one by one \(\eta\alpha\pi\sigma\sigma\sigma\tau\eta\iota\alpha\iota\nu\sepsilon\nu\iota\tau\ieta\tau\iota\nu\ \alpha\pi\epsilon\iota\rho\omicron\ \kappa\alpha\tau\eta\ \eta\epsilon\kappa\alpha\tau\omicron\tau\omicron\nu\) in a finite time. For there are two ways in which length and time are called ‘infinite’: they are called so either in respect of divisibility or in respect of their extremities. So while a thing in a finite time cannot touch things quantitatively infinite, it can touch things infinite in respect of divisibility: for in this sense the time itself is also infinite; and so we find that the time occupied by the passage over \(\delta\iota\epsilon\nu\omega\alpha\iota\) the infinite is not a finite but an infinite time and that the touching \(\eta\alpha\pi\tau\epsilon\sigma\sigma\tau\eta\iota\alpha\) of the infinities is made in times not finite but infinite in number.\(^{14}\)

Aristotle interprets Zeno as making the claim that it is impossible for one to traverse or touch an infinite number of points \textit{in a finite time}, since the completion of that task would require an infinite amount of time. He attempts first to attack the Dichotomy by distinguishing infinite divisibility from infinite extension and then accusing Zeno of ignoring the parallel between the natures of magnitude and time insofar as both can be infinite either in division or extension. Aristotle’s point is that one can traverse an infinite sequence of points \textit{in a finite time} because both a finite magnitude and a finite span of time are infinitely divisible, and because of that there will be as many instants within a given finite span of time available for completing the task of touching as there are points on a finite given surface to be touched.

This reply to Zeno’s Dichotomy is adequate but it overlooks the deeper significance of the paradox.\(^{15}\) Acknowledging that time is infinite raises the further question of how one can endure a finite stretch of time if every stretch of time is composed of an infinite sequence of

\(^{14}\) 233a21–31.

\(^{15}\) Ross 73–4.
instants.\textsuperscript{16} Nevertheless, the interpretation of the Dichotomy that Aristotle offers us is telling. For he rewords what the Dichotomy is trying to disprove, i.e., traversing an infinite number of points (and thus moving from $B$ to $E$), in terms of touching points one by one in a finite time. This rewording suggests a reading of the Dichotomy that is different from the first one examined above. For touching one by one an infinite number of points in a finite time amounts to performing an infinite number of discrete tasks one after another in a finite time, and this way of stating the matter is crucially different from supposing the track to be of infinite length.

The second reading of the Dichotomy is also suggested in a passage from the pseudo-Aristotelian treatise On Indivisible Lines in which the author posits a magnitude that is without parts (τι μεγετησ αμέρεσ) in response to Zeno’s Dichotomy:

Furthermore, in accordance with Zeno’s argument there must be a magnitude that is without parts (τι μεγετησ αμέρεσ), if indeed it is impossible to touch (ηαπσαστηαι) infinite points in a finite time, touching them one by one (κατη' ηεκαστον), and if it is necessary for anything that moves to arrive at the halfway point first, and if there is a halfway point in everything that is not without parts.\textsuperscript{17}

In this passage, the author attempts to defend the existence of motion, thus corroborating the second reading of the Dichotomy, by rejecting the assumption in Zeno’s Dichotomy that motion requires traversing a continuous, infinitely divisible, magnitude. We may elaborate on the author’s train of thought in the passage above as follows. Motion would indeed be impossible, in accordance with Zeno’s Dichotomy, if it were the case that magnitude is continuous, for then in order to move across the continuum one would have to touch an infinite number of points one after the other, which is impossible. But in fact motion is

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\textsuperscript{16} Aristotle himself realized that this is so, see Phys. 263a18–22.
\textsuperscript{17} 968a18–22.
\end{flushright}
possible; motion does not require traversing a continuous magnitude because magnitude is not continuous, i.e., there is a magnitude that is without parts \((\tau \text{ μεγετηοσ αµερεσ})\) at which division must cease.

Both Physics and On Indivisible Lines suggest that Zeno maintained that motion across a continuum amounted to performing an infinite number of discrete tasks one after another and that motion is impossible because it is impossible to complete such a task. Therefore, let us accept the second reading of the Dichotomy as the correct one and interpret it thus:

(1) If one moves from \(B\) to \(E\), when one reaches \(E\) one has touched one by one an infinite number of points in a finite time.

(2) It is impossible to have touched, in a finite time, all the points in an infinite sequence of points, i.e., an infinite number of points.

(3) Therefore, one cannot move from \(B\) to \(E\).

Premise (2)

Much can be said, and has been said, both for and against this argument, but I will pursue a line of criticism that differs from the standard one that most modern commentators take. Most modern commentators reject Zeno’s argument by rejecting the above Premise (2), claiming that it is impossible to perform an infinite number of tasks.\(^{18}\) However, this way of assessing the paradox is misguided, for it can be shown that Zeno has a specific conception of activity in mind in (2), which compels us to take it as a true premise. To be sure, the way in which (2) is stated above is ambiguous because it may ultimately mean either that it is impossible to perform a last task in a sequence of tasks that has none or that it is impossible for a state of affairs to obtain in which all of the tasks have been performed but not a last task (the first is true and the sec-

\(^{18}\) For instance, see Barnes 273 and Long 148.
ond is false). However, two features appear in the Greek which suggest that what Zeno intends to be at issue in (2) is the possibility of completing the performance of an infinite number of discrete tasks one after another. The first important feature is the phrase κατη' ηεκαστον, which appears both in the Physics and several times in the treatise On Indivisible Lines. This phrase suggests that the points are to be gone through one after another, but it still leaves the manner in which they are to be gone through unsettled. However, the second important feature of the Greek, the appearance of the aorist ηαπσαστηαι, does allow us to settle the manner in which the points are to be taken, for the usage of the aorist in the Greek suggests that the type of touching in question is some kind of discrete activity, like tapping. If Zeno had interpreted motion as a continuous activity, like stroking one’s finger across a smooth surface, the present infinitive ηαπσαστηαι would have been used in the place of the aorist infinitive in the texts. Thus, although the way in which (2) is stated in English is ambiguous, a thorough examination of the relevant Greek texts (Phys. 233a21–31 and On Indivisible Lines 968a18–b4) suggests that Zeno intended (2) to deny the possibility of completing the performance of an infinite number of discrete tasks one

19 Barnes 268.
20 Many commentators tend to overlook this phrase and the appearance of the aorist ηαπσαστηαι in constructing their own formulations of what appears above as (2): Barnes 263, Kirk 270, Long 145.
21 233a23.
22 968a21, b1, b2.
23 That is to say, this phrase on its own leaves it open whether, when one touches an infinite number of points one after another, the act of touching is a smooth, continuous motion, like gliding one’s finger across a smooth surface, or a series of discrete acts of touching, like a series of taps. I thank Professor Gill for her comment that eventually helped me to realize the ambiguity of the phrase κατη' ηεκαστον.
24 This is why I think that Barnes’ illustration using a perfect cube (Barnes 263) is misguided; if Zeno had interpreted motion as a continuous activity, the present infinitive ηαπσαστηαι would have shown up in the Greek. I have benefited from discussion about the force of the aorist with Dr. Floyd.
after another.\footnote{The appearance of the present infinitives διεναι and ἡπαστήσαι at Phys. 233a30–31 may seem to cause trouble for my interpretation of (2). However, that is not the case, because the part of the passage in which the present infinitives appear is irrelevant to the matter of how one should construe Zeno’s Dichotomy, since in that part of the text Aristotle is putting forth his first solution to the paradox, not restating Zeno’s argument; he does that at 233a 23–4, and it is very telling that the aorist infinitives διελθεῖν and ἡπαστήσαι appear there.}

If motion from B to E really amounts to completing the performance of an infinite number of discrete tasks one after another—which has yet to be proven—then motion from B to E is impossible, since it can be shown that it is impossible to complete an infinite sequence of discrete tasks one by one. In fact, a passage from the treatise On Indivisible Lines, which immediately follows the passage discussed above, provides us with a lead to establishing this conclusion:

Now if something moving on a line touches an infinite number of points in a finite time; and if something moving more rapidly traverses more than something slower in an equal amount of time, and the movement of thought is the most rapid, then thought could touch upon an infinite number of things one by one in a finite time; and so if counting is thought touching one by one, it is possible to count an infinite number of things in a finite time. If this is impossible, there must be an indivisible line.\footnote{968a23–b4, my italics.}

In the italicized phrase, the author assumes that there is some close connection—the precise nature of which is indeterminate—between performing an infinite sequence of discrete tasks one by one and counting. If we assume that attempting to count one by one all the positive natural numbers (1, 2, 3, etc.,) is an instance of attempting to complete an infinite sequence of tasks one by one, then we can easily show that it is impossible to complete an infinite sequence of tasks one by one. We can never complete the task of counting all the members of the infinite sequence of natural numbers one by one, because no matter what number we arrive at there will still be another one greater. The
same argument applies more generally to any case of attempting to complete an infinite sequence of tasks (including the task of touching the infinite sequence of points on a track one by one): no matter how close we get to “finishing” our tasks there will always be another task to perform.  

Premise (1)

Let us now leave premise (2) aside and consider premise (1). In this premise, Zeno assumes that moving from $B$ to $E$ is equivalent to completing the performance of an infinite sequence of discrete tasks one by one, and we shall find that this assumption is what is ultimately responsible for the Dichotomy’s perplexing character. But before presenting our own solution to the paradox, let us briefly examine Aristotle’s second attempt to solve the Dichotomy, for it carries greater insight than most scholars realize.

After stating his reservations for his first resolution the Dichotomy, Aristotle states his second solution to the problem:

And so we must reply to the man asking whether it is possible to traverse infinite things (either in time or distance) that in a way it is but in a way it is not. For if they exist in actuality, it is not possible, but if potentially, it is; for someone moving continuously ($συνεχοσ$ $κινουμενοσ$) has traversed infinitely many points accidentally and not without qualification. For it is incidental to the line to be infinitely many halves, but its essence and being are different.

Aristotle frames this reply to Zeno’s Dichotomy in terms of his distinction between potentiality and actuality; it is possible, he thinks, for one to traverse an infinite number of points, on the condition that they exist potentially, but not if they exist actually. This distinction, in turn, corresponds to a distinction between continuous motion and the performance of a series of discrete movements and amounts to the

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27 For an argument that makes a similar point by slightly different means, see Long 146.
claim that motion is possible, if one travels continuously to \( E \), but not if one attempts to actualize the infinite number of points that exist potentially on the way to \( E \). This would be tantamount to performing an infinite number of discrete actions one after another, which he presumably, and rightly, thinks is impossible.\(^{29}\)

The most important aspect of Aristotle’s second reply to Zeno’s Dichotomy for our purposes is the implicit distinction between continuous (\( \sigmaυνεχοσ \)) activity and the performance of a number of discrete acts, for the awareness and acceptance of this distinction is what allows us to dissolve the paradox. Premise (1) of Zeno’s Dichotomy illicitly imitates an instance of a continuous action, i.e., continuous motion from \( B \) to \( E \), to the performance of an infinite sequence of discrete actions one after another; it describes motion from \( B \) to \( E \) as the performance of an infinite number of discrete acts of touching one after another. This imitation is the main source of the Dichotomy’s profundity, for once one thinks of motion as the completion of an infinite sequence of discrete acts, it becomes hopeless for one to imagine how motion across a continuum could be possible. We have already shown that it is impossible to complete the performance of an infinite sequence of tasks when one performs them discretely, one after another. However, once one acknowledges the obvious difference between continuous motion and the performance of discrete actions one after another, the haze surrounding the possibility of motion disappears; moving from \( B \) to \( E \) is no longer a paradoxical and puzzling notion.\(^{30}\)

Admittedly, a true Eleatic, like Zeno, would not immediately find this reply to the paradox convincing, since he might say that we are ulti-

\(^{29}\) Aristotle appears to maintain that a sufficient condition for actualizing a point is performing some discrete action at the place where the point exists potentially, e.g., stopping at it.

\(^{30}\) This reply, as one might be able to tell, is characteristically Wittgensteinian. In fact, it is quite similar to Wittgenstein’s dissolution of Augustine’s puzzle about the possibility of measuring time by pointing out that it arises as a result of his (illicit) assimilation of measuring of time to measuring lengths (26). However, while Wittgenstein almost always gives a diagnosis of what led to the postulation of the puzzle in the first place, I shall forgo attempting this in the case of Zeno’s Dichotomy out of fear of making claims that are too speculative.
mately relying on our senses to verify the possibility and existence of both discrete and continuous motion and the difference between those two types of motion. But, at least, a tentative reply is at hand. For in solving the Dichotomy by making the distinction between continuous motion and the performance of discrete actions, we are not dogmatically denying that motion is impossible, like Antisthenes the Cynic, who tried to refute the paradox by standing up and taking a step. Rather, because we are elucidating a conceptual distinction that the paradox blurs, we are quite in keeping with the Eleatic command to “judge by reason.”

Conclusion

Despite our efforts to persuade them of our solution of the Dichotomy, the Eleatics might still say that this conceptual distinction is grounded in experience, or find some other reply, and remain dissatisfied with our way of handling the paradox. While we could, in turn, insist that the conceptual distinction is perfectly valid albeit grounded in experience, it is likely that they would not be satisfied with that answer either, or any other one that makes experience in one way or another an important aspect of the reply. Of course, this does not show that our solution of the paradox is wrong. Rather, the mere fact that we may construct so many replies in line with the spirit of Eleaticism shows that Eleaticism is something more than merely the sum of its arguments; like the Hydra, as soon as one of their arguments is beaten into submission, another one or two or three is bound to take its place immediately. In order to shatter the illusion of profundity surrounding the Dichotomy for an Eleatic once and for all, we would have to engage in a thorough criti-

31 Cf. Elias’ In Cat. 109.20ff.
32 “…judge by reason the strife-encompassed refutation spoken by me” (Kirk 249, cf. Sophist 242a1–2).
cal examination of the roots of Eleaticism, starting with Parmenides. While we do not have the time nor space to attempt such a Herculean task in this paper, we have, at least, provided a possible topic for future philosophical scrutiny.

References