The Possibility of the Existence of an Actual Infinite

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IN A WORK entitled *The Mormon Concept of God*, evangelical Christians Francis Beckwith and Stephen Parrish criticize the Latter-day Saint (hereafter LDS) concept of God. Their philosophical arguments that the LDS concept of God contains philosophical flaws are based mainly on Mormon eternalism—the idea that individuals, matter, space, and time have always existed. Their central criticism stems from an argument by William Lane Craig that it is impossible for an actual infinite number of things to exist in the real world. I intend to show that the argument from the impossibility of an actual infinite is fallacious, and that Beckwith and Parrish neither understand the nature of LDS doctrine nor argue effectively that it is incoherent.

First, I will assess the philosophical arguments surrounding the actual infinite. Then, I will discuss several scientific arguments that are relevant to the discussion of whether time could have begun. Finally, I will conclude that Beckwith and Parrish's work is ineffective in discrediting the LDS understanding of God.

There are numerous statements by LDS leaders that support the claim that an actual infinite is included in the LDS view. To support this view, Beckwith and Parrish cite Joseph Fielding Smith, Heber C. Kimball, and Bruce R. McConkie—each a general authority in the LDS church (54). For example, Heber C. Kimball explains that "we shall go back to our Father and God, who is connected with one who is still farther back; and this Father is connected with one still further back, and

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so on" (19). It is hard to imagine that any consistent LDS view could avoid referring to an actual infinite.

The Existence of an Actual Infinite

In *Theism, Atheism, and Big Bang Cosmology*, William Lane Craig and Quentin Smith debate whether there was a beginning of time and whether God caused it. Craig's argument that the universe had a beginning centers on the impossibility of an actual infinite (4). He claims that since an actual infinite is impossible, the universe must have begun to exist. Beckwith and Parrish rely mainly on Craig's arguments to support their objections to LDS eternalism. They also attack other instances of an actual infinite that they claim are part of the LDS view.

Craig offers four arguments against the possibility of an actual infinite, two philosophical and two scientific. The philosophical arguments attempt to refute the idea of an actual infinite by reducing the idea to an absurdity. I challenge Craig's *reductio* refutation on two grounds: first, Craig shows that, at best, an actual infinite is counterintuitive, not that it is impossible; second, the counter-intuitive inferences that Craig draws from the idea of an actual infinite are a result of his faulty conceptualization of it.

"Infinity" can mean a variety of things. A "potential infinite" refers to a magnitude that can be indefinitely divided or extended. This is the infinity of Zeno's paradoxes and is also called a "variable finite" by Cantor. An "actual infinite" represents the number of all the numbers in $\{1, 2, 3, . . .\}$. It is not *becoming* infinite; it is a complete, deterministic whole.

Craig begins by emphatically asserting that the existence of an actual infinite cannot be inferred from the well-defined mathematical concept alone. Craig says Cantor "won for the actual infinite the status of legitimacy that it holds today" (6). However, after acknowledging that the mathematical concept of an actual infinite is a generally accepted idea, Craig is quick to point out that it does not necessarily follow that it ontologically exists. Indeed, even the mathematical integrity of an actual infinite has been doubted, which makes its ontological instantiation even more unlikely.

Craig's argument does not assert that the existence of an actual infinite is *impossible*, only that its possibility cannot be directly inferred

from the coherence of the mathematical concept alone. For this reason, I only consider his argument that the assumption of an actual infinite produces absurdities to be significant in drawing the conclusion that an actual infinite cannot exist in the real world. Craig's first argument may be summarized as follows:

1. An actual infinite cannot exist.
2. An infinite temporal regress of events is an actual infinite.
3. Therefore an infinite temporal regress of events cannot exist.

Both Aristotle and Aquinas have disputed premise two, claiming that beginningless time is actually infinite (Craig 25). They held that the past was only potentially infinite since only the present moment actually exists. Craig, however, feels this analysis is inadequate: "the fact remains that since past events, as determinate parts of reality, are definite and distinct and can be numbered, they can be conceptually collected into a totality" (25). For this reason, Craig concludes that a beginningless past necessarily entails the existence of an actual infinite.

If the conditions just quoted are all that is required to constitute an actual infinite, the future must either be an actual infinite or must come to an eventual end. Craig contends that the future differs from the past because it is not actual, while the past is already realized. However, it seems that Beckwith and Parrish cannot consistently maintain this position, for they hold that God is both timeless and omniscient. As such, the future is already actual to God and thus certainly can be conceptually collected into a totality. Therefore, Beckwith and Parrish must either allow for the existence of an actual infinite number of future events, or accept that time will end at some point.

Perhaps Beckwith and Parrish would be willing to concede that time will end. In that case, they may successfully avoid this problem and consistently maintain their position that an actual infinite is impossible. However, it makes humanity's future look pretty bleak (since no individual can ever be permanent or immortal), unless a person can somehow be removed from time. Regardless, there are other reasons for believing that Craig's argument against the actual infinite is inadequate.

Craig supports premise one of the above argument by saying: "While the actual infinite may be a fruitful and consistent concept in the mathematical realm, it cannot be translated from the mathematical

world into the real world, for this would involve counter-intuitive absurdities" (9). The rest of this section will discuss this claim.

Craig demonstrates absurdities that arise by attempting to perform arithmetic operations on an actual infinite. For example, Craig proposes a library with an infinite collection of books as an example of an actual infinite in the real world. The collection becomes no larger in number after adding another infinite collection of books. No matter what happens at the circulation desk, no matter how many books are checked in or out, the library will almost always keep an infinite number of books on its shelves. The only exception is when certain infinite combinations of books are checked out—but even checking out an infinite number of books could leave the library full or empty depending on the circumstances. Adding a new book to the library causes other problems. Every natural number has already been used to identify an already included volume, so there is none left for additional books! After citing dozens of similar examples, Craig draws the conclusion that these absurdities sufficiently support his claim that an actual infinite is impossible.

Craig also mentions Hilbert's Hotel, a notable illustration of an instantiated infinite by David Hilbert (13). In this example, there are an infinite number of hotel rooms rather than infinite books in a library, but the absurdities drawn are all analogous. Imagine that every room in the hotel is occupied. Surprisingly, moving every current guest down the hall a room or two can always accommodate more guests. There will always be enough rooms for the current guests and an infinite number of new ones as well.

Virtually every example Craig and Hilbert offer can be simplified to just one central absurdity: the number of objects in an actual infinite never changes no matter what kind of finite mathematical operation is performed on it. In these simplified terms, their examples offer no more absurdity than that which follows from the mathematical definition of infinity itself. Zermelo-Fraenkel axiomatic set theory defines an infinite set as one that is equinumerous with some of its proper subsets (Craig 8). Indeed, the definition itself seems counter-intuitive.

A proper subset is a subset that does not have every member of the original set. For example, the set {2, 4, 6, . . .} is a proper subset of {1, 2, 3, . . .} because the second set includes members not in the first. The odd thing is that *every* member of the first is in the second, and yet they have

the same number of members! This is strangely contrary to Euclid's maxim that the whole is greater than the parts. We know these two sets must have the same number of members because the Principle of Correspondence states that if the members of the two sets can be drawn into a one-to-one correspondence then they are equinumerous. Craig eventually clarifies that his absurdities all relate to the Principle of Correspondence, which is indispensable in the definition of infinity (24).

There is certainly something strange about a set that can have proper subsets equinumerous with itself, but this strangeness is straightforward in the definition of infinity. If Craig's collection of books were divided in half by removing every second book, and afterward it contained fewer books than it did before, *that* would be an absurd result according to set theory. Then we would have to say that the set must not have been infinite at all. If it were infinite, by definition it would have to have still been equinumerous with the proper subset formed by removing every second book.

John Mackie has argued that an actual infinite's strange properties fail to make it unbelievable (Craig 98). For example, the impossibility of accommodating a new book with its own unique natural number is not absurd. Whether a new book can be numbered with a natural number is irrelevant since it can always be distinguished in some other way, labeled and included in the collection. There are infinitely many real numbers available to accommodate any new book at any sequential position in the collection.

Other absurdities cited by Craig stem directly from the counter-intuitiveness of reversing Euclid's maxim. In an actual infinite, the whole is actually *not* greater than a part. It is a surprising result that an infinite number of books behaves so strangely, but in principle it is not necessarily an impossible result. In general, humans have neither experience nor intuition regarding the infinite. Without experience to verify what is or is not possible (or what is "absurd") when dealing with infinity, Craig's examples may only verify that examples of an actual infinite in daily life are just as counter-intuitive as the mathematical infinite. Although *creating* an infinite collection of books is impossible, predictable results (although strange) following from the assumption that one exists do not demonstrate impossibility.

Beckwith and Parrish anticipate the argument that an actual infinite is not absurd because its properties are clearly expected from its definition.

However, in their rebuttal they commit a straw man fallacy. They compare the defensive claim that the actual infinite's strange properties are not absurd to an apologetic for square circles (67). Square-circle apologists say that a thing that is internally contradictory *by definition* cannot be faulted for not being internally consistent. They offer no compelling reason that we should accept the validity of square circles while expecting us to disregard our basic intuitions about reality. However, although Beckwith and Parrish fail to acknowledge them, there may be especially compelling reasons in LDS theology to grant that an actual infinite is possible.

In philosophy, accepting an unlikely proposition without evidence is usually not allowed. However, in religion, faith allows for and necessitates acceptance of incompletely supported propositions. If an actual infinite is truly implied by LDS doctrine and if the actual infinite obeys a different set of counter-intuitive rules than finite sets, accommodations in the LDS world view can be justifiably made. Further, the inadequacy of limiting all existing entities (including time and space) to a finite magnitude seems greater than the stretch of intuition required to allow for an actual infinite.

I have shown that it is not clear whether an actual infinite is absurd. Regardless, absurdity is not sufficient for impossibility. Craig's examples only verify that counter-intuitive results are achieved from arithmetic manipulations. This point is anticipated by the definition of infinity, and should be expected since infinity is not an intuitive concept in general. For these reasons, Craig's first argument fails to prove the impossibility of either an actual infinite or beginningless time.

Craig's second philosophical argument is that an actual infinite can never be formed by successive addition. However, Smith answers this thus: "although [an infinite series] can never be completely synthesized in a finite time, it can be completely synthesized in an infinite time" (Craig 89). It appears that Smith is willing to concede that successive addition can never form an actual infinite if the process has a beginning or if time is finite. Only in the case of infinite time is an actual infinite possible from successive addition. Craig contends that Smith begs the question in his reply by assuming infinite time. However, Smith's argument only shows that the concept of an actual infinite is self-consistent. Craig can be just as readily accused of question begging by requiring an

initial assumption of finite time in his argument about successive addition.

As previously shown, many absurdities suggested by Craig stem from the faulty manner in which the infinite is understood or described (for example, conceptually imagining a beginning to infinite successive addition). These absurdities do not reflect as much on the possibility of an actual infinite as they emphasize errors in the way in which it is conceptualized. They tell us what can and cannot be done with an actual infinite, and they prompt us to make distinctions in our conceptual understanding. There are other examples of this as well. For instance, Craig imagines a finite person removing every second book in an infinite library by a successive process. Such a task is impossible if one concedes that successive addition to an already begun process is impossible. Absurdities derived from some concept involving an actual infinite may show the concept itself to be problematic without telling us anything about the possibility of an actual infinite. An infinite library and an infinite hotel are impossible from the outset, but not because they are infinite. They are impossible because they must have always existed to exist at all, and because infinite successive changes in them (like shifting rooms or checking out infinite amount of books) are impossible.

Other objections to an actual infinite refer to an *infinite value*. It is a mistake to conceive of infinity as a number because this definition invites one to project unfounded numeric qualities onto it without even realizing it. Every specific number is finite, and when infinity is called a number merely because it defines a mathematical relationship, the issue becomes ambiguous. This fact is important because it clarifies arguments involving reference to the "infinitieth" member of an infinite set (Mackie 93). There is no such member, and absurdities stemming from it are nonsense.

There are no past moments infinitely distant, and a traversal of the infinite must never have begun, but always have been. Projecting finite conceptions on infinity (like a furthest point or a beginning of traversal) is likely to produce absurdities. However, such absurdities say nothing about the infinite itself, merely the misconception with which it may be held.

To review, the absurdities Craig draws from the concept of an actual infinite are merely counter-intuitive results, not impossibilities or self-contradictions. Furthermore, all genuine problems in the concept

of a real actual infinite can be understood as errors in mixing up infinite and finite processes, operations, attributes, etc. Craig has yet to show that any impossibility is entailed in a correctly characterized infinite. Therefore, whether an actual infinite is possible remains an open question.

Craig does offer scientific evidence that the universe must have begun, but the theories he cites to defend this thesis are based on scientific methods and theories, with all the accompanying limitations. I will now discuss Big Bang Cosmology in order to refute Craig's latter arguments. Without compelling scientific support, both Craig's scientific and philosophical arguments for time's beginning are inconclusive.

The Problem of Science and Religion

Beckwith and Parrish cite Craig's scientific arguments to support their claim that an actual infinite is impossible and that the LDS concept of God is incoherent. However, it is a mistake to use science in a discussion of religious coherency since science never makes necessary claims about truth, and can therefore never definitively show the impossibility of a particular point of view. Despite this fact, science would still have evidential value to the current discussion if it were not the case that LDS cosmology seems to deny its fundamental assumptions. Many inconsistent results can be derived by combining two systems with conflicting assumptions. Big Bang Cosmology offers a perfect example of why a discussion involving both science and religion can easily suffer from fallacious reasoning.

The Big Bang Theory is the most widely recognized scientific explanation of the origin of the universe. Tracing time backwards, all matter in the universe converges on one point. This point is a singularity with neither spatial nor temporal extension. According to the Big Bang Theory, it was from this point that everything in our universe originated.

This theory relies on at least two very important assumptions. First of all, it assumes that there must be uniformity throughout time and space except in rare, temporary instances. This means that the physical laws science observes are true everywhere and at all times. This assumption is consistent with experience accumulated during mankind's short history, but more importantly, this assumption is justified because theories based on it have been successful in making accurate predictions about the universe.

The second fundamental assumption science makes is that the universe behaves naturalistically. That is, all phenomena can be explained in terms of natural laws and causes without referring to a supernatural agent. It can be understood as minimally allowing for a mechanistic, determinate understanding of the universe independent of whether God is the one who created it.

Without the assumptions of uniformity and naturalism, it is difficult to imagine how scientific theories could be formulated and what purpose they could serve. The ultimate statement that these assumptions make about the universe is that it is ordered and predictable (within a certain scope). Although scientific theories may not be able to predict all experimental outcomes, their capability and limits are understood in most cases. When theories are unable to make definitive predictions, they often establish probabilities that have just as much value.[1] Without being able to rely on the continuation and integrity of natural laws, society could not function the way it does. Therefore, for the purposes of science, the assumptions of naturalism and uniformity are not only convenient but also necessary, regardless of whether they are ultimately true.

The Big Bang Theory makes verifiable predictions and explains otherwise puzzling data. For example, the red shift of light from other galaxies, background microwaves bathing the universe, and increasing entropy are all explained by the Big Bang Theory. Few other current theories explain such a large amount of data. However, the Big Bang Theory has severe limitations as well.

In 1970, Stephen Hawking and Roger Penrose, two prominent theoretical physicists, demonstrated that a space-time singularity was necessarily included in the Big Bang Theory (66). A singularity is an infinitely small point with a very large mass—in this case, the mass of the entire universe. The density at this point is infinite because the dimensions are infinitely small. Although mathematics has shown that space-time singularities are unavoidable if we accept Einstein's theory of relativity, science cannot determine the properties of such a singularity.

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[1]The Heisenberg uncertainty principle guarantees that not all experimental outcomes are predictable. However, quantum mechanics uses probabilities to make predictions where absolute calculations are impossible (Hawking 81).

General relativity and all other physical laws fail to hold at the space-time singularity (Hawking 157).

Some assume that this means physical events and time are possible before the Big Bang—we would just be unable to determine what the universe would be like before the singularity, and anything in the former universe would be irrelevant to us. However, this is not the case. Quentin Smith summarizes why the definition of a singularity precludes this possibility.

> Furthermore, it belongs analytically to the concept of the cosmological singularity that it is not the effect of prior physical events. The definition of a singularity that is employed in the singularity theorems entails that it is impossible to extend the space-time manifold beyond the singularity. The definition in question is based on the concept of inextendible curves . . . if there is some point p beyond which it is possible to extend the space-time manifold beyond which geodesics or time-like curves can be extended, then p by definition is not a singularity. (Craig 120)

In other words, it is built into the mathematics that the singularity is an absolute beginning of time, not just an unpredictable anomaly in our naturalistic universe. With no prior events leading to it, this singularity seems to have been either caused by a timeless God, or not caused by anything. Either way, it did not have a cause in conventional time.

If the universe was *uncaused* then it must be self-existent or have spontaneously come into being from nothing. The latter option, according to Craig, runs contrary to "the strongest support experience affords" (121). Granting this, we must admit that *many* things about the Big Bang run contrary to experience (no prior time, infinite density, breakdown of all natural laws)! Paul Draper contends that any criticism of the idea of uncaused creation stems from acceptance of the metaphysical intuition, "something can't come from nothing" (47). Draper notes that metaphysical intuitions are notorious for being false. However, if something can come from nothing, it seems that the universe must necessarily be a very arbitrary place indeed. To be consistent with science's assumption of naturalism, we must assume that there are non-arbitrary reasons why the universe came to be. Therefore, the proposition that

the universe began uncaused seems inadequate for science as well as religion.

Craig quotes Fred Hoyle, a contemporary scientist: "Hoyle realizes that an absolute beginning of the universe points beyond the universe to a reality more ultimate than itself, since to say it simply sprang into being for no reason out of nothing is 'unsatisfactory'" (45–46). It is unsatisfactory because it denies the assumption of naturalism. Craig draws from this some support for the notion that God must have created the Big Bang. He criticizes Hoyle for his fruitless pursuit of a "steady state" theorem against all odds and even after most scientists had already given up. Indeed, Craig claims that Hoyle's work was motivated by a desire to avoid the theistic implications of a reality more ultimate than the universe.

Actually, such implications may be avoided without contradicting the assumption of naturalism. There is a version of the Big Bang Theory that includes the idea that the universe is self-existent. Stephen Hawking, who first influenced the scientific community to accept space-time singularities, afterward decided that he did not believe that singularities were real. He believes that space-time is finite, yet has no boundary or edge. Rather, its curvature becomes greater and greater until, in effect, it folds back on itself. If one were to travel in one direction toward the edge of the observable universe, one would eventually arrive back at one's starting place. This sounds ridiculous, yet has surprising support:

> Thus the no boundary proposal is a good scientific theory in the sense of Karl Popper: it could have been falsified by observations but instead its predictions have been confirmed. . . . All the complicated structures that we see in the universe might be explained by the no boundary condition for the universe together with the uncertainty principle of quantum mechanics. (180)

Hawking does not feel that this entails the existence of God. In the case of the no boundary proposal, the universe is both finite and boundless. Its existence is in need of no further explanation—it *just is*.

However, there is a much simpler and more intuitive method of understanding these issues. It is easy to forget that the assumptions of uniformity and naturalism, on which we base so much of our thinking, are only assumptions. LDS scripture actually seems to contradict or at least

qualify them. In LDS theology there are references to different "orders" of time,[2] ultimate laws outside the scope of science,[3] and spiritual influences unexplained by science[4]—each of which is an exception to naturalism. Religious people already accept a spiritual reality more fundamental than that suggested by naturalism. It should not be too much of a stretch to understand that man's theories—both scientific and religious—are not infallible for this reason.

Theories about the origin of the universe are ultimately weak. "Unless and until we achieve an understanding of the Planck epoch [the period prior to 10^{-43} seconds after the big bang], it is hardly more than scientific bravado even to speculate about the origin of the universe. It may well be that this question is beyond science. Still, we cannot resist" (Hawley 459–60). The limitation of science results because the assumption of uniformity becomes increasingly less likely as time approaches the Big Bang.

If scientists are compelled to admit that uniformity may break down at certain points in the past, scientific theories must then be limited to a particular scope of time. For this reason, no scientific evidence is ultimately compelling in denying the existence of infinite time. Because in an LDS view any apparent properties of our temporal universe may merely be part of a mortal "sphere" of existence (D&C 93:30), I have now shown that there are both scientific and religious reasons for disregarding Craig's scientific arguments.

To conclude, Beckwith and Parrish fail to show in *The Mormon Concept of God: A Philosophical Analysis* that the LDS view is internally incoherent. Although they do show contemporary notions of the actual infinite to be counter-intuitive, they do not demonstrate the existence of an actual infinite to be impossible. The LDS world view is capable of

[2]For example, Abraham was shown relative *orders* of planets and time (Abraham 3: 3, 4, 9). Joseph Smith has also observed that past, present and future are all "one eternal 'now'" to God (220).

[3]Considering the LDS view that God *became* God (Smith 345), he must have done so by obeying moral laws prior to himself.

[4]The description of the "light of Christ" in D&C 88: 7–13 refers to a type of influence on the physical world unacknowledged and unexplained by the physical sciences.

accommodating counter-intuitive conclusions because of its dynamic doctrinal structure. Science and philosophy can be interesting in evaluating religious positions, but are subject to the common limitations of all academic human endeavors—they are limited by a mortal paradigm. The LDS view of eternalism is internally coherent, although it may not be readily compatible with traditionally held beliefs and assumptions about God or the nature and origin of the universe.

Works Cited

Beckwith, Francis J., and Stephen E. Parrish. *The Mormon Concept of God: A Philosophical Analysis.* New York: The Edwin Mellen Press, 1991.

Craig, William Lane, and Quentin Smith. *Theism, Atheism, and Big Bang Cosmology.* New York: Oxford University Press, 1993.

Hawking, Stephen. *The Illustrated A Brief History Of Time.* New York: Bantam Books, 1996.

Hawley, John F., and Katherine A. Holcomb. *Foundations of Modern Cosmology.* New York: Oxford University Press, 1998.

Kimball, Heber C. "Temple And Endowments—Raising Grain And Building Storehouses—Dedication." *Journal of Discourses* 4.19 (1856): 224-27.

Pojman, Louis P. *Philosophy of Religion: An Anthology.* New York: Wadsworth Publishing Company, 1998.

Smith, Joseph, Jr. *Teachings of the Prophet Joseph Smith.* Ed. Joseph Fielding Smith. Salt Lake City: Deseret Book Press, 1938.