

The Nature of Logical Constants

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Characterizing the distinction between the “logical” and “non-logical” expressions of a language proves a challenging task, and one with significant implications for the nature and scope of logic itself. It is often claimed that logical truths are statements that are “true by virtue of form,” and, likewise, that arguments are logically valid because of their respective “forms,” not because of their contents (Sider 2).¹ Take, for example, a straightforward piece of reasoning:

(P1) Maria is in Berlin or she is in Vienna.

(P2) Maria is not in Berlin.

(C) Maria is in Vienna.

If this argument is valid—which, of course, in classical logic, it is—then it is typically held to be valid because of the *structure* of the argument and not because of certain material facts about the world. So it seems as if we should

¹Contrast the idea of *truth preservation by virtue of form* with the idea of an argument that is truth-preserving by virtue of the *meaning* of certain terms:

(P1*): Nick is a bachelor.

(C*): Nick is an unmarried man.

We might think that the truth of (P1*) guarantees the truth of (C*), but it is not by virtue of the *form* of the argument; it is by virtue of the meaning of the expressions “bachelor” and “unmarried man.”

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be able to create an abstract formulation into which we could uniformly substitute expressions for the non-logical names and predicates as they appear while still maintaining a truth-preserving chain of reasoning:²

(P1) $A(x) \vee B(x)$

(P2) $\sim A(x)$

(C) $\therefore B(x)$

This all seems well, good, and familiar. But in creating a more abstract version of our initial argument, further questions arise: for one thing, why is it that we elected to keep the expressions “or” and “not” fixed, but left open the possibility for substitution of “Maria,” “Berlin,” and “Vienna”? And is there a principled way to decide which expressions are “logical”—and thus must remain *constant* under various substitutions in order to give certain schemas their logical forms—and which are not? As Peacocke points out, it is not enough to merely say that the logical constants are the terms whose substitution might affect the validity of the argument—such an explanation would be circular, leaving us to define validity in terms of logical constants and logical constants in terms of validity. An adequate definition of logical constants must be one that *determines* which arguments are valid, not the other way around. It is only once we define the logical constants that we develop conceptions of logical validity and logical consequence—that is, conceptions of what conclusions are licensed given certain premises (Peacocke 222). As the logician and mathematician Alfred Tarski asserts following his explication of the concept of logical consequence, “at the foundation of our whole construction lies the division of all terms of a language into the logical and extralogical” (“On the Concept of Following Logically” 188). It seems as if we have good reason to think that our concepts of logical validity, consequence, and truth—that is, the realm of logic—are grounded in the nature of logical constants.

In §1, I will provide an exposition of two prominent strategies for defining logical constants: first, the “permutation invariance” tactic as advanced by Tarski, and second, the “grammatical particles” approach as proposed by Quine. In §2, I will consider the objection that logic should be understood as *relative*, and, accordingly, there is no real debate to be

²For simplicity's sake, I am relying here on a Quinean notion of logical consequence. To put it briefly: c is a logical consequence of $(p1, p2 \dots)$ if and only if there is no way to uniformly substitute expressions for nonlogical expressions in c and $(p1, p2 \dots)$ so that $(p1, p2 \dots)$, all become true but c does not. (See Quine, *Philosophy of Logic* 58–60).

had about the nature of logical constants. According to this objection, there is no such thing as a “logical constant” *simpliciter*, and the task of trying to define them as such is inevitably a fruitless one (MacFarlane, “Logical Constants” 43). In §3, I will argue that, even supposing relativity in the realm of logic, there remains an alternative motivation for defining logical constants: logic plays an enormous *normative* role in reasoning, and we ought to have a firm grasp on the concepts that govern our thought, speech, status as “rational” beings, and so on. Thus, regardless of logic’s objectivity (or lack thereof), there remains an important impetus for coming to some sort of collective agreement about the logical constants, even if such an agreement recognizes the wholly *constructed* nature of the constants themselves.

§1 Tarski and Quine

Tarski defines logical “notions” as those expressions whose extensions are “invariant” under “all possible one-one transformations of the world onto itself” (“What Are Logical Notions?” 149). This proposal is known as the *permutation invariance* definition of logical constants. To make Tarski’s suggestion clearer, imagine a domain D , $\{x,y,z\}$: a *permutation* is a function that would assign to each member of D one member of the same domain, and to any member of the domain a *different* member, thereby making the function one to one, e.g., assigning x to y , y to x , and z to z . Now imagine the largest possible set—the set of all objects in the world O , $\{o_1, o_2, o_3, \dots\}$.³ As professor John MacFarlane explains, the important concept at play is that an expression is *permutation-invariant*, and, resultantly, logical, “just in case its extension on each domain of objects is invariant under all permutations of that domain” (“Logical Constants” 20). Logical expressions’ extensions are invariant under “all possible one-one transformations of the world onto itself” (as Tarski requires) because they remain the same no matter which arbitrary permutation of O we take.

The central intuition seems to be that logical expressions are “indifferent to the particular identity of objects”—in a sense, they are topic-neutral (MacFarlane, “What Does it Mean to Say Logic is Formal?” 72). On this account, logic abstracts from the specific nature of objects or meanings of words and is instead concerned with general facts about the world (73). For example, consider domain D again and the existential

³Note that, unlike modal conceptions of logical consequence, this theory deals solely with objects in the actual world.

quantifier “ \exists ,” whose extension is the set of all non-empty subsets of D . No matter what arbitrary permutation of D we take, \exists 's extension will remain the same; the set of all non-empty subsets of D is invariant. In contrast, imagine the same domain and instead the true sentence “ Rxz ,” where “ R ” is the two-place “cousin” relation. Suppose that Rxz is true just in case x is the cousin of z , and suppose that x is the cousin of z , but z is unrelated to y . Now take an arbitrary permutation $f(p)$ that maps x to y , y to x , and z to z . The extension of the Rxz is *not* permutation invariant, because, as illustrated with $f(p)$, the sentence might well turn out to be false; $f(p)$ makes it so that we would need y to be the cousin of z in order for Rxz to be true, and we have already stipulated that y is unrelated to z . Unlike the extensions of logical notions, the extension of the cousin relation depends on specific empirical features of the objects to which it refers. Felicitously, on the permutation-invariance theory, the identity predicate, the standard truth-functional connectives (\sim , $\&$, v , \supset), and the universal and existential quantifiers qualify as logical constants.

Quine, on the other hand, defines logical constants in terms of *grammar*. He contends that an argument belongs to the realm of logic only if it “hinge[s] purely on the structure of the sentences concerned, rather than depending on content,” and further, that “the structure of sentences consists in grammatical constructions” (“Grammar, Truth, and Logic” 17). Thus, logic is concerned with “the truth constructions that hinge solely on grammatical constructions” (Quine, “Grammar, Truth, and Logic” 17). As MacFarlane puts it, the logical constants of a language are the language's grammatical particles—that is, the expressions by means of which complex sentences are composed—as opposed to the simple, nonlogical expressions of which atomic sentences are constructed (“Logical Constants” 7). Thus, the atomic sentences “Maria walks” and “Maria talks” contain no grammatical particles, but the more complex sentence “Maria walks and talks” contains the grammatical particle “and.” It is particles like “and” that comprise the logical constants.

Logic and grammar are *so* closely tied for Quine, in fact, that logical form is what grammatical form *becomes* when “grammar is revised so as to make for efficient general methods of exploring the interdependence of sentences in respect of their truth values” (“Grammar, Truth, and Logic” 21). The connection between grammatical structure and logic is special not because there is something special about grammatical particles *per se* (after all, we could come up with some arbitrary language in which grammatical particles had no connection at all to logical constants), but because we *ought* to employ a language in which grammar is a reliable guide to truth-conditions (MacFarlane, “Logical Constants” 8). There is

an important *pragmatic* role that logical constants play. For Quine, “ \sim ,” “ $\&$,” and the existential quantifier qualify as logical constants.

§2 The Challenge from Relativity

At this point, we have seen two ways in which philosophers have attempted to define logical constants. Both methods are controversial, but they hold a shared aim. That aim is to distinguish the logical expressions from the nonlogical ones, and, as a result, to demarcate the realm of logic. Importantly, though, Tarski and Quine readily admit that logical constants can only be defined relative to how they are used within a certain formal language or context. Tarski even explicitly *rejects* the idea that there exists “a proper, true meaning of a [logical] notion, something independent of actual usage, and independent of any normative proposals, something like the Platonic idea behind the notion,” calling such an idea “strange” and “foreign” (“What Are Logical Notions?” 145). Given these qualifications, one might reasonably wonder whether the entire debate about logical constants loses its significance. After all, if logical constants can only be defined in the context of a certain language and its speakers, and thus have no claim to objectivity,⁴ then it might be the case that disagreement about logical constants amounts to nothing more than mere differences in intuitions about *logical consequence*, i.e., differences in intuitions about what follows logically from what. There simply is no such thing as a “logical constant” *simpliciter*. To illustrate: recall the differences between Quine’s and Tarski’s theories about whether “ \vee ,” “ \supset ,” and “ \forall ” are logical constants; perhaps these debates (and still further ones about modal and temporal operators, etc.) simply do not *have* a single correct answer. It seems that the definitions of logical constants are more context-sensitive than we might like them to be, and that the context might even be so narrow as to include only one person or community’s logical commitments, so long as those commitments are internally consistent.

I have formulated the challenge from relativity somewhat vaguely thus far, but there are various ways that we could flesh out the objection in greater detail. Philosopher of mathematics Michael Resnik, for example, holds that logic is not even “cognitive,” and that “statements of logical truth, validity, and the like do not state facts” (Shapiro 4). It follows that there is no fact to the matter about logical constants—for if there were, then there would be facts about logical truths and validity

⁴I understand “objectivity” roughly as mind-independence.

(e.g., assuming the facts about “ \sim ” accepted in classical logic, we get the logical truth “ $\models \sim \sim p \supset p$ ”).⁵ As such, from the argument that logic is not *cognitive*, it follows that logic is not *objective*; if logic is not cognitive, then there simply cannot be any logical facts to which we assign objective truth or falsity.⁶ Similarly, philosopher Crispin Wright holds that statements of logical necessity are “not apt for objective truth or falsity” (Shapiro 4). He claims that statements can express genuine facts “only if there are contexts ... in which it is *a priori* that differences of opinion concerning one of the relevant statements can be explained only by disclosing... some material ignorance, error, or prejudice” (Wright 200). According to Wright, it is doubtful that logic provides one of these contexts. Indeed, two logicians could each be perfectly free of ignorance, error, and prejudice, and yet still be in disagreement about whether or not a statement represents a logical necessity. Recall once more the differences in Tarski’s and Quine’s theories: on Wright’s picture, neither Tarski nor Quine is correct, *per se*; they just happen to have conflicting views. Here again we see an argument for irrealism in the realm of logic—there are no mind-independent, objective facts about whether a proposition represents a logical necessity. In fact, Wright sees a striking similarity between disagreements about logical necessity and disagreements about which jokes are humorous. This view renders logic, and the nature of logical constants, primarily a matter of personal taste (Wright 206).⁷

If the nature of logical constants is *not* objective and is instead context-dependent, and if that context is determined by the intuitions about logical consequence that are already held by the very philosophers who are aiming to define the logical constants, then perhaps those who debate the nature of logical constants are merely talking past one another. The challenge from relativity suggests that disagreements about logical

⁵ “ $\models \sim \sim p \supset p$ ” states a proposition or logical *fact*; i.e., it is cognitive, contra Resnik’s original premise. So negation can’t be a logical constant. We could run this line of argument with any of the logical constants.

⁶ There is a helpful analogy to be drawn here with metaethical noncognitivism: similarly to what Resnik holds about logical statements, most metaethical noncognitivists hold that moral statements (e.g. “murder is wrong”) do not express judgments or propositions, and thus cannot be true or false. Instead, moral judgments are mere expressions of *attitudes*, which are not truth-apt.

⁷ Logical necessity is a relevant concept here because it is quite natural to think of “truth by virtue of form” as *entailing* a concept of logical necessity—after all, if an argument is truth-preserving by virtue of its form, then *necessarily* if the premises are true, the conclusion is true. As such, logical truths are statements that are *necessarily* true by virtue of form—that is, they can be derived from the empty set. So, assuming we accept this conception of logical necessity, rejecting logical necessity amounts to rejecting the idea that there is such thing as “truth by virtue of form” via *modus tollens*.

constants are analogous to disagreements about subjective values. Moreover, if the competing notions of logical constants are really derived from competing notions of consequence from the outset, then logical constants ought to simply be relativized to different notions of logical consequence (MacFarlane, “Logical Notions” 43). For example: we might have various definitions of the material conditional depending on our notion of logical consequence: \supset_{N-M} for non-monotonic logic, $\supset_{N-M'}$ for a slightly different flavor of non-monotonic logic, \supset_R for relevance logic, \supset_I for intuitionistic logic, and so on. Some of these definitions might overlap and some might come apart from each other, but no definition is “better” *per se* than any other, so long as they each accurately reflect the logical commitments of the system within which they are defined. If I believe that we are not licensed to draw conclusions from defeasible generalizations, but my peer thinks that we are, then we may have different views on the definition of “ \supset .” But, importantly, this conflict simply arises because of differing intuitions about logical consequence.

Even though there may in fact be formal distinctions between the logical and nonlogical expressions within different logical systems, such distinctions do not amount to anything particularly philosophically interesting, because we have no principled way of deciding which logical system is best. If logic is relative, as Resnik and Wright argued, then there is no objective fact of the matter about whether (e.g.) nonmonotonic logic is better than classical logic, and, as such, no fact of the matter about how best to define logical constants. So, arguably, the *real* debate ought to be over the nature of logical *consequence*, and the so-called “nature” of logical constants will simply emerge as a by-product of that discussion. Here, then, we arrive back at one of the original problems mentioned in the introduction: we wanted our logical constants to determine our notions of logical consequence and *not* the other way around. But if we reject an objective view of logical constants, we have no choice but to admit that the nature of said constants is likely derived from our pre-held intuitions about logical truth and consequence.

§3 Logic as Normative

Let us take stock. We are now at a point where the question of the “nature” of logical constants seems totally misguided. If there are no objective facts about logical constants, then it is wholly unclear whether we could properly demarcate them even if we wanted to. If the disagreement about logical constants really just amounts to disagreement about preconceived notions of logical consequence, then logical constants

are not even the proper objects of discussion here. So the challenge from relativity is twofold: (i) there is no objective “nature” of logical constants, and, accordingly, (ii) different notions of logical constants ought to simply be relativized to different notions of logical consequence, thereby eliminating the need for debate over logical constants in the first place. But is this challenge really compelling? What of the questions posed in the introduction, which made it seem evident that determining the nature of logical constants is *essential* if we wish to determine the nature of good arguments? In this section, I will argue that the *normative* role that logic plays in reasoning provides impetus enough to define the nature of logical constants, challenges from relativity notwithstanding.

Imagine that you are visiting a community that accepts a *material* notion of logical validity; whenever a premise is false or the conclusion is true (in the actual world), the argument is valid. Thus, *every* expression in the language is a logical constant—that is, *every* expression in a given argument gives the argument its logical form. Consider the following arguments:

(P1) Chicago is north of Lexington.

(C) London is south of Cambridge.

(P1) Hillary Clinton won the 2016 election.

(P2) Whales are mammals.

(P3) The USA was an Allied force in World War II.

(C) Madeline Albright lives in a retirement community on the Moon.

(P1) The Sun orbits the Earth.

(C) $2 + 2 = 4$.

These arguments are materially valid, since either a premise is actually false, the conclusion is actually true, or both. Admittedly, material validity fails to capture certain *modal* notions that other classical and non-classical logics are better equipped to handle. But those in the material validity community *are* able to correctly assert that their arguments are truth-preserving. You are left wringing your hands, unable to quite figure out *how* to argue against material validity, given that you do not believe that there is an objective “fact to the matter” about logical constants.

I would like to suggest that it is the *normative* role that logic plays in reasoning that results in our instinctive rejection of material validity. MacFarlane elaborates a bit on said role: he explains that we often form a link between logical validity and “evaluation and criticism of reasoning,” and that, given a clear consensus on validity, we could “transpose questions about logical validity into questions about how we *ought to think*” (“In What Sense (If Any) Is Logic Normative for Thought?” 3; emphasis mine). Logic provides the *norms* for thought and reasoning—in fact, such is the *principal* (and perhaps *only*) difference between logic and other areas of investigation (e.g. empirical sciences). Logic is a *tool*, and we engage in it with the purpose of learning what we ought to believe (5). Here, then, is where we should identify the central motivation for defining the nature of logical constants. *Regardless* of whether there is an “objective” nature of logic, and regardless of whether a relativist picture could simply index different concepts of logical constants to different concepts of consequence, there *remains* the fact that we often make evaluative judgments with specific notions of logical constants and consequence at play. It is not as if we simply accept a relativist standpoint and thus accept just *any* chain of reasoning as a valid inference, even if that chain is internally consistent and perhaps even *materially* valid. Reasoning *well* amounts to revising one’s beliefs as one ought to (4), and therein lies the central problem with material validity: material validity allows us to jump from premises to conclusion with seemingly no reasoned connection between the two.⁸ Our theoretical commitments about the objectivity or relativity of logical constants need not play any role in our evaluation of them; our believing that their nature is relative does not commit us to believing that all methods of defining them are just as good as any others.

We can find other reasons for which material validity fails to adequately capture logical reasoning: take, for example, hypothetical reasoning. As Oxford professor Ian Rumfitt points out, an argument cannot be materially valid if it “supports reasoning from a supposition that might be false” (11). According to Rumfitt, this type of counterfactual reasoning is *essential*; in many cases of intuitively sound arguments, “there is no convincing explanation that does not at some point advert to our ability to apply our deductive capacities to suppositions,” i.e., to claims that may well be false (8). It is this type of reasoning that is most valuable, allowing us to *extend*

⁸Of course, so too do some classically valid argument forms (e.g., arguably, the principle of explosion). This is perhaps a reason to question classical validity, but it is no reason at all to accept material validity.

our knowledge and generate notions of necessity and possibility by “tracing the implications” of things we cannot be said to know (7). Such extensions of knowledge require modal notions, which material validity cannot accommodate. For example: the counterfactual “If I were to leave my house at 7:00, I would arrive at 7:30” requires a concept of *necessity* in order to be true; thus, statements like these cannot be used in material reasoning. Here, then, we are able put forth an argument against material validity with no reference to “objectivity” in the domain of logic. Our claim is a normative one: proper logical reasoning *ought* to allow us to gain knowledge that we could not otherwise obtain through mere sense perception.

Thus, in a sense, the debate over logical constants is best understood as a *metalinguistic* one: even if we accept that there is no objective nature to logical constants or concepts, we still might disagree over how we ought to use the phrases “logically valid,” “logical truth,” “logical consequence,” etc. We have normative reasons to favor certain uses of these phrases over others, and, consequently, we have normative reasons to favor certain uses of the phrase “logical constants” over others. Defining the logical constants will allow us to differentiate the logical from the non-logical, even if it is only within a certain context. Given wide enough scope, the idea that logical constants are context-dependent is not so troubling after all; if the context is wide enough to include, say, the vast majority of human communities on this planet, then the fact that logical constants have no mind-independent nature is of no practical significance to us. Even if there is nothing mind-independently *objective* about these normative reasons—perhaps, for example, they merely have arisen for pragmatic or evolutionarily-driven purposes—the context of this normativity is so wide that its context dependence simply does not matter. Of course, this proposal casts doubt on our ability to gain logical knowledge *a priori*. But the upshot is that the charge from relativity is left destabilized, because *regardless* of logic’s objectivity or lack thereof, the normative nature of logic gives us sufficient motivation to define logical constants.

§4 Conclusion

As we have seen, there are numerous ways that we might define logical constants—I briefly surveyed the proposals from Tarski and Quine, but that list is far from exhaustive. Both of those proposals are subject to an objection from relativity, according to which there simply is no fact to the matter about the nature of logical constants. The central threat from relativity is this: there simply is no real debate to be had about the nature of logical constants. I contend, however, that even assuming relativity, the

debate over logical constants is not a trivial one; there is a serious issue over what logical constants are. This is because there is a central *normative* role that logical constants play in our everyday thought and reasoning, and, as such, arriving at a satisfactory definition of them (even one that is context-dependent) remains an important task.

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