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The Identity Problem in *Ante Rem* Structuralism and an Objection to Its Legitimacy

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Structuralism is a theory in the philosophy of mathematics that presents mathematics as the study of certain types of relationships and claims mathematical objects are essentially their structural properties. This doctrine leads to a problem about mathematical identity which is particularly apparent in Stewart Shapiro's *ante rem* structuralism on account of a rather strong constraint, called the "faithfulness constraint," that the philosophy of mathematics should be descriptive.

I will first briefly describe *ante rem* structuralism. Then, I will outline the development of what has become known as the identity problem, particularly the metaphysical concerns Jukka Keranen raises. Next, I explain a possible response to the identity problem first posited by Christopher Menzel who argues that the metaphysical constraints and concerns raised by Keranen are not an accurate interpretation of mathematics. This would allow *ante rem* structuralists to reject the identity problem as it improperly describes mathematics, violating the faithfulness constraint. Finally, I will put forth my own objection to the identity problem that, on account of the faithfulness constraint, all *ante rem* structuralism must say is that which mathematicians say regarding the metaphysics of mathematical identity. This will lead to a sort of philosophical quietism about mathematics, yet I will show that this is compatible with *ante rem* structuralism.

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1. Structuralism as a Theory of the Philosophy of Mathematics

In contrast to most traditional philosophies of mathematics such as platonism or nominalism about mathematics which argue that mathematics is the study of the abstract or concrete objects of mathematics, respectively, it does not matter what the specific objects of mathematics are to structuralists (Linnebo 11 and 102). Rather what matters is the relations exemplified between the objects. For example, 2 apples and 2 apples is 4 apples and 2 + 2 = 4. The actual objects in the examples above are different, but the relationship itself remains the same, namely the addition relation. Similar to the abstract-concrete debates about mathematical objects in traditional philosophies of mathematics, structuralism has debates about the reality of structures. The realist camp of structuralism is called non-eliminative, while the more nominalist camp is called eliminative. Non-eliminative structuralists admit structures as real and, typically, abstract objects while eliminative structuralists deny the existence of any real structures. This taxonomy is far from a clear black-and-white, and there are many intermediate positions (Reck and Schiemer section 1.2).

Ante rem structuralism admits structures as real and abstract objects which exist independent of any mathematics (Shapiro, *Philosophy* 89). It also has a rather complicated ontology that we address in greater detail later, but, briefly, the objects of mathematics are whatever could be in a mathematical relationship with another object. Any collection of objects that stand in some mathematical relationship to each other we may call "systems." Then, to pick out just the relational properties of the systems, we let a "structure" be a real and abstract object which consists only of the relations exemplified by a system regardless of the actual objects of the system (Shapiro, *Structure* 146–147).

Shapiro helps motivate *ante rem* structuralism with what he calls "the faithfulness constraint," which says that philosophy should not change how mathematics is done or thought of by mathematicians (*Identity* 289). The point of the faithfulness constraint is to ensure the philosophy of mathematics is felicitous with mathematics itself. Mathematicians do not typically talk of their practice being imaginary or made-up,[1] so, according

[1] Insofar as he may be considered a representative of how mathematicians consider mathematics to be, Penrose says, "Yet we shall find that complex numbers, as much as reals, and perhaps even more, find a unity with nature that is truly remarkable. It is as though Nature herself is as impressed by the scope and consistency of the complex-number system as we are ourselves, and has entrusted to these numbers the precise operations of her world at its minutest scales" (Penrose 73).

to the faithfulness constraint, we must not say that the objects of study are not real as the eliminative structuralists do. Similarly, mathematicians talk of numbers as singular objects which may be expressed both independently and in relations to other mathematical objects. So, philosophy must follow suit and do the same. Structures can be motivated by noticing that mathematics does not particularly care what specific objects are being discussed so long as the relationships are maintained.

It is tempting to say that mathematical objects are real and abstract, but if we say this, we are then charged with the difficult task of finding a convincing story of how we can know of them and use them. Rejecting platonist principles and saying that mathematical objects are fictional or concrete may lead us to say that mathematical claims to some extent are dependent on our minds and are difficult to communicate and apply or are even entirely subjective (Frege 16–17). Additionally, this runs counter to our common intuition about mathematics. This dilemma about the metaphysics and epistemology of mathematical objects was first raised by Paul Benacerraf, and so is known as Benacerraf's dilemma (662).

Structuralism responds to this dilemma by adopting a relative ontology of mathematics. A system exemplifying a mathematical structure will have objects whose relationships to each other are the same as the relations between the relata of the structure. These relata are known as places in a structure. Places are real and abstract objects; however, their function is context dependent. They may be viewed as offices that are filled by objects that exemplify a mathematical structure or, in other contexts, may be viewed as real and abstract objects themselves (Shapiro, *Philosophy* 82–83). Places are essentially their structural properties, or relations to other places (169). For instance, all there is to the initial place in the natural number structure (i.e., 0 in arithmetic, a system that exemplifies the natural number structure) is its being the additive identity ($n + 0 = n$) and its being the successor to nothing. The identity problem argues that this thesis about places in structures is problematic.

2. The Identity Problem

2.1 Commitment to Structural Properties in the Identity of Indiscernibles

Shapiro says, "If we are to have a theory of structures, we need an identity relation on them" (*Philosophy* 91). An identity relation on structures is any formula which is sufficient to determine if two mathematical objects are identical. Keranen specifically requires that an account of identity fulfill two criteria: there must be a way to determine whether any two objects denoted by singular terms in the domain of discourse are identical and the account must apply for all objects in the domain of discourse (312). Since mathematical objects are essentially their structural properties, an account of identity which satisfies these requirements is:

$$\text{(II)} \quad \forall xy(x = y \leftrightarrow \forall F(Fx \leftrightarrow Fy))$$

where F is some structural property (316, and Burgess 287). We may follow the literature and refer to II as the identity of indiscernibles for mathematical objects.[2] Following this, let a and b be structurally indiscernible if all and only all the structural properties of a are structural properties of b.

Here is Keranen's argument that *ante rem* structuralism must commit to II (314):

Structuralism's Commitment to II

(1) If the properties of mathematical objects are exactly their structural properties, then all and only all objects which are structurally indiscernible are identical.

(2) The properties of mathematical objects are exactly their structural properties.

(3) Therefore, all and only all objects which are structurally indiscernible are identical.

The support for the first premise is as follows: both Shapiro (*Philosophy*, 91) and Keranen (314) claim on the basis of a doctrine of Quine (23) that structuralism must furnish an account of identity for mathematical objects.[3] For the account of identity to satisfy the two requirements listed above, it must follow the schema $\forall xy(x = y \leftrightarrow –)$ where the – is replaced with an expression that says that all and only all the relevant properties of x and y are the same. This schema ensures that any two mathematical objects can be substituted for x and y, it will allow us to determine if they are identical, and it applies to all mathematical objects regardless of if they can be denoted by a singular term (Keranen 313). If mathematical objects only have their structural properties, then the schema is completed with an expression about structural properties, giving us II.

[2] For example: Menzel 84; Shapiro, *Identity* 286; Keranen 31

[3] Though interesting and perhaps objectionable, this point is too large to consider in further detail here.

The support for the second premise is as follows: mathematical objects are places in structures (Shapiro, *Philosophy* 77). Since there is nothing more to structures than particular relations whose relata are those places, the places in structures have only their structural properties (72). This commitment leads to some problems.

2.2 The Problem with II

There are cases of distinct[4] objects which appear to have the exact same structural properties, thus making them indiscernible from and equivalent to each other. Perhaps the most famous example is that of the imaginary numbers i and $-i$. I will follow the convention in the literature and let $i = -j$ and $j = -i$ in order to remove ourselves from the purely linguistic idea that i and $-i$ have relationships to 0 similar to that of the integers (complex numbers are neither positive nor negative).[5] To fully realize the structural properties of i and j, we must begin with a short introduction of the complex field.

A mathematical field is a simply a set of objects and relations on them, typically + and ·. The complex field **C** is simply the set of all complex numbers C and the relations + and · which are defined by the additive and multiplicative identities $z + 0 = z$ and for any complex number $z \cdot 1 = z$. **C** has the numbers i and j such that (Burgess 288):[6]

$$(R) \qquad i^2 = -1 \wedge j^2 = -1 \wedge i + j = 0.$$

This complex number system of ordinary mathematics exemplifies the complex number structure, meaning all the relational properties of the objects of the complex number system are structural properties of the places in the complex number structure (Shapiro, *Philosophy* 89–90). So, the relational properties of i and j which are captured by R are the structural properties of the places in the complex number structure which i and j are filling in. R is sufficient to describe all the structural properties of i and j.

According to II, if i and j have exactly the same structural properties, then they are identical. We will now show that i and j appear to have the same structural properties. The form of any complex number z is $a + bi$ where a and b are any two real numbers. Let us now consider a mapping called

[4] When I say two objects are distinct, I mean that mathematicians talk of them as nonidentical, not that they are structural indiscernible.

[5] See: Menzel, Burgess.

[6] $i^2 = -1$ and $j^2 = -1$ function as shorthand for multiplication expressions.

"complex conjugation," which takes a complex number $a + bi$ and maps it to its complex conjugate $a - bi$. The complex conjugate of i is j and vice versa. Let the result of applying complex conjugation to the complex field **C** be the field **C**′. **C**′ will have all the same elements as **C** and the additive and multiplicative identities are unaffected. Additionally, we stipulated that **C** has two numbers i and j with specific structural properties captured by R. Under complex conjugation, R becomes

$$(R') \quad j^2 = -1 \wedge i^2 = -1 \wedge j + i = 0$$

in **C**′. Even after applying complex conjugation, i and j retain the same structural properties in R' as in R, namely i and j are still the roots of −1 and additive inverses of each other. After mapping every element in **C** to its complex conjugate, the structural properties of i and j are unchanged, i and j have the exact same structural properties. From this, II entails that $i = j$, a clear violation of the faithfulness constraint.[7]

Complex conjugation is a nontrivial automorphism over **C**. It is an automorphism because it is an isomorphism (which is a mapping of one structure to another which preserves the structural relations and properties) which maps its domain to itself. It is nontrivial because it does not map every element to itself. This example is generally true: for a field with a nontrivial automorphism, there are objects in that field which share all and only all their structural properties (Keranen 323). By II, all objects which are mapped to each other under a nontrivial automorphism must be identical to each other. This leads to some mathematically contradictory expressions.

[7] There is some technical nuance to be fleshed out. **C** is a mathematical ring since it has a multiplicative identity $z \cdot 1 = z$. This makes complex conjugation a homomorphism, which is a mapping of one structure to another that preserves the multiplicative identity. (Isomorphisms are a type of homomorphisms. The distinction does not concern us here but it must be noted that complex conjugation is also an isomorphism.) **C** has the relations + and · that range over all of its elements, and since complex conjugation is a homomorphism, they are preserved in **C**′. To put this in the language of structuralism, the relationship every number in **C** has to every other number is the exact same after being mapped to **C**′ by complex conjugation. Since there is for every element in **C** an element in **C**′ that has the exact same relationships, namely its complex conjugate, by II the two elements are identical (Leinster 2). This, of course, would mean every number is identical to its complex conjugate. We may introduce another algebraic relation such as <, which could order the real numbers and be sufficient to distinguish them from each other (i.e., $\neg\forall x(Fx \leftrightarrow Fx^*)$ where x^* is the complex conjugate of x and x ranges over the real numbers). Complex conjugation would no longer preserve the structure of **C** for the real numbers, but, because there is no ordering of complex numbers, i and j would not benefit a distinguishing property. In fact, there is no algebraic relation that could be added to **C** that would produce a relationship from i to some other number that would not also be true of j (Burgess 288). In other words, complex conjugation shows that i and j have exactly the same structural properties ($\forall F(Fi \leftrightarrow Fj)$), which entails $i = j$ by II.

2.3 The Nature of Structural Properties

A structuralist might argue that there are some structural properties, called haecceities, that can only apply to one object (e.g., the property of being identical to 2), alleviating the identity problem (Keranen 313). However, Keranen argues that this move is not available to structuralists. To introduce haecceities as structural properties, the structuralist would need a way to pick out a singular term denoting a place in a structure. If we take an existing system that exemplifies a structure to express haecceities by utilizing the objects in the system to denote places in structures, then places in structures will have properties that depend on there being a system that exemplifies the structure (316–317). This would violate a core notion of *ante rem* structures: "Structures exist whether they are exemplified in a nonstructural realm or not" (Shapiro, *Philosophy* 89).

In similar fashion, if we were to stipulate a language of singular terms to denote places in structures to express haecceities, then the objects of systems exemplifying the structure would have structural properties that relate the objects of the system to the structure itself. This would disallow the system to exemplify the structure because haecceities would express a relation a place in a structure has to itself, but in the system it would express a relation between the object filling in a place in a structure and the corresponding place. Again, this would violate a core principle of *ante rem* structuralism because there would be no way to accurately abstract structural relations from systems exemplifying a structure (Shapiro, *Structure* 146). Haecceities, no matter how they are construed, seem to be against the spirit of structuralism. These two constraints force structural properties to be only the properties which can be specified without use of individual constants (Keranen 317).

3. Analysis of the Plausibility of Individuating Formulas

3.1 Preliminaries

Perhaps the most powerful response to this problem is a more mathematical approach objecting to the notion that distinct objects have a structurally individuating formula (a structural property that an object has exclusively), which would challenge the use of structural properties as criteria for identification. Let L be a language, or a set of constant symbols and non-logical predicates like + and ·. Let a system $\boldsymbol{A}$ that utilizes L as its language be an L-system. Let A be the domain of $\boldsymbol{A}$ that is the (non-empty) set of objects of the system, and let V be the interpretation of L that maps every constant in L to an element in A and every predicate in L to a relation over the set A. The L-system $\boldsymbol{A}$ is denoted by the pair $<A,V>$. Arithmetic over the natural numbers is an L-system where L is the set of all the natural number numerals (0, 1, 2, etc.) and the arithmetic 3-place predicates + and ·, A is the set of the actual places in the natural number structure, and V takes the numerals and arithmetic predicates + and · and maps them to unique objects in A and relations over A.

The complex field **C** has a similar language L_c as the natural number language, but it includes all the real numbers as well as the complex numbers i and j. V_c takes the predicates + and · and maps them to unique relations over A_c (which is the set of all the places in the complex number structure). We noted before that the structure of **C** is the same regardless of the order of i and j in R. This means that in the L_c-system $\boldsymbol{C}<A_c,V_c>$ every formula expressed by the predicates + and · that is true of i is also true of j. Formally, let the type of $a \in L$, $tp(a)$, be the set of all formulas φ expressible in L which are mapped to relations over A that are true only of a. So, $tp(i)$ in the L_c-system $\boldsymbol{C}$ is the set of every true formula φ expressed in L_c that is mapped to every relation over A_c that is true of i alone. The demonstration from earlier that there is a nontrivial automorphism on **C** entails $tp(i) = tp(j)$. Our definition of indiscernibility can be altered to work with this notation: a and b are indiscernible if and only if $tp(a) = tp(b)$. Substituting this into II gives: $a = b$ if and only if $tp(a) = tp(b)$. Conversely: $a \neq b$ if and only if $tp(a) \neq tp(b)$. That is, there is some relation (an individuating formula) that is satisfied by a but not by b.

3.2 An Objection to the Commitment to II

The goal of this response to the identity problem is to show that the requirement for nonidentity that there be an individuating formula will lead to us having to admit additional formulas specific to certain objects which will turn out not to be structurally dependent on specific objects the formulas pertain to, or, in Menzel's terms, this requirement will lead to over-specification (99). As it stands, there is no individuating formula for i and j in $\boldsymbol{C}$, so we must expand $\boldsymbol{C}$ in such a way that the structural properties are preserved to admit individuating formulas. The expansion of $\boldsymbol{C}$ is called $\boldsymbol{C}'< A_c,V_c'>$. Consider a formula in $\boldsymbol{C}'$ that individuates i: $x^2 = -1 \wedge x = -j$. Clearly, this is true of i and i alone, and all the structural properties of **C** are preserved, thus $tp(i) \neq tp(j)$ which entails $i \neq j$. However, when we apply the automorphism of complex conjugation to $\boldsymbol{C}'$—call the result $\boldsymbol{C}''$—the formula used to individuate i becomes $x^2 = -1 \wedge x = -i$, which no longer individuates i, but rather j. So the expansions we used to individuate i

are isomorphic under complex conjugation, which means the individuating formula is not actually a structural property of ***C*** (100).

Objection to the Commitment to II

(1) If II, then all distinct objects have an individuating formula.

(2) There are some distinct objects that do not have an individuating formula.

(3) Therefore, II cannot be the case.

The support for the first premise is as follows: If II is the case, then identical objects have exactly the same structural properties. Inversely, distinct objects do not have the same structural properties. In other words, for distinct objects *a* and *b*, it is not the case that all and only all structural properties of *a* are structural properties of *b*. If it is not the case that distinct objects have the same structural properties, then there is some property that one object has that no other object has. By definition, that property is an individuating formula. So, if II is the case, then distinct objects have an individuating formula.

The support for the second premise is as follows: *i* and *j* are distinct objects. There are no algebraic properties that can individuate them in **C** (Burgess 288). To supply individuating properties, we must expand **C** to allow it to express individuating formulas for *i* and *j*. However, each expansion of **C** that individuates *i* is isomorphic to every expansion that individuates *j*. So expansions of **C** that claim to individuate *i* and *j* suffer from the same problem that motivated the expansion in the first place. Since isomorphisms are structure preserving, this reveals that *i* and *j* are not able to have different structural properties, even though mathematicians speak of them as distinct. So there appear to be a certain class of objects that are distinct yet structurally indiscernible.

4. An Objection to II

I argue the identity problem comes from a misunderstanding of equality in mathematics: "=" seems to make a statement about the denotations of the terms on either side. In other words, $x = y$ means the things which x and y stand in for are the exact same thing, not just that they are substitutable or structurally indiscernible. Whatever is written down on either side of "=" is part of some language *L*. An *L*-system has a distinct domain *A* which consists of the actual objects of the structure. So, using the language of *L*-systems, $a = b$ expresses the identity of the interpretation of *a* and *b*.

Let us return to the complex numbers *i* and *j*. The inequality $i \neq j$ means that $V(i)$ and $V(j)$ are not the same place in the complex number structure, not necessarily that they are structurally discernible. This is because structural indiscernibility is not what makes objects identical in mathematics. Rather, two objects are identical because there is some mathematical algorithm that can be done on one object to arrive at the other object. For instance, a division algorithm can be carried out on a fraction to arrive at the decimal equivalent. In other words, mathematics determines the interpretation of terms.

So, structural indiscernibility does not seem to be sufficient for identity. *i* and *j* have the same structural properties, which the faithfulness constraint was introduced to address and was evidenced by complex conjugation, but they are still distinct because mathematics does not interpret them identically. Because of the faithfulness constraint, structuralists do not need to provide philosophical grounds for the distinction between *i* and *j*. They only need to explain their place in mathematics and the complex number structure. Additionally, structuralism's focus on structural properties does not imply that structural properties are related to identity. A mathematician will wholeheartedly admit that every instance of *i* could be replaced by *j* and vice versa without any alteration in meaning, but this implies only that *i* and *j* have the same structural properties, not that they are equal. Complex conjugation does not prove $i = j$ but rather that complex numbers are defined by a sole expression whose solutions may be written however we desire, and the mere fact that there are exactly two solutions to that relationship is enough to distinguish the two solutions (Shapiro, *Identity* 287). I argue that structural indiscernibility does not appear to have a role in mathematical identity.

My Objection to the Commitment to II

(1) If "=" in mathematics asserts the identity of the interpretation of both terms on either side of it, then II cannot be the case.

(2) "=" in mathematics asserts the identity of the interpretation of both terms on either side of it.

(3) Therefore, II cannot be the case.

The support for the first premise is as follows: though "=" has a substitutive force, it asserts that the objects denoted by the terms flanking the equal sign are identical. The denotations are determined to be identical by mathematical processes and algorithms, not by philosophy. *Ante rem* structuralism seems to be able to make this move on account of a strong interpretation of the faithfulness constraint. So, if "=" asserts the interpretation of the terms flanking it, then a principle that makes something other than there being a mathematical algorithm to get from one term to the other be sufficient for identity, like II, cannot be the case. The support for the second premise is as follows: nonidentical terms can have the same interpretation and be equal. For instance, 3/2 is not the same term as 1.5, yet they are interpreted to the same object in the set of places in the rational number structure, namely that place which is as many places after the first place as before the second place.

There is an obvious objection: it seems as if we are making identity and distinctness into a rule we let mathematicians dictate as they see fit. That is, it seems unclear how i and j are interpreted to distinct objects when all of their relevant properties under *ante rem* structuralism are the same. We almost seem forced to say that i and j are distinct because mathematicians say so and point to the fundamental theorem of algebra as evidence,[8] so the notion of identity seems quite dubious for structuralism if we reject II. However, this objection simply comes from a misunderstanding of what structuralism sets out to do.

Structuralism need not unravel the metaphysics of places because, under the faithfulness constraint, mathematics does not explicitly comment on its own metaphysics besides a strong implication of reality and abstractness. Imaginary numbers are no exception and one of the stronger exemplifications of mathematicians' silence about metaphysical questions. All mathematicians seems to say about imaginary numbers is that i and j are the two guaranteed solutions to the polynomial $x^2 = -1$. So, since *ante rem* structuralism takes the objects of mathematics to be places in structures, there must be two places in the complex number structure that have this structural property. *Ante rem* structuralism has no charge to say anything more illuminating about i and j or the complex number structure. This makes structuralism rather weak in explanatory power, but with that come many desirable consequences, including that it does not challenge mathematical claims on philosophical grounds.

[8] The fundamental theorem of algebra guarantees that there are exactly two solutions to the polynomial $x^2 = -1$.

5. Conclusion

Structuralism need not comment on the metaphysics of mathematics that deeply. Though perhaps somewhat unsatisfying, under the faithfulness constraint, structuralism is obligated to refrain from commenting much on identity. Mathematics generally presupposes identity, being defined implicitly by statements of primitive properties. Structuralism, by the faithfulness constraint, must follow suit. Structuralists are not able to say why there are two distinct but structurally indiscernible roots of –1, but, by the faithfulness constraint, that is all structuralism sets out to do.

The faithfulness constraint is a strong and very useful doctrine available to *ante rem* structuralists. It allows them to accurately describe mathematics and respond to difficult problems in other theories about the philosophy of mathematics at the expense of further philosophy. In this paper, I outlined structuralism, then I detailed the identity problem and Keranen's metaphysical concerns. Afterwards, I explained Menzel's response to the identity problem before offering my own objection to the legitimacy of the identity problem as a puzzle for *ante rem* structuralists. Structuralism is a unique philosophy of mathematics and, as I have shown here, it has a striking ability to support, on philosophical grounds, the results of mathematics.

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