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Indispensability and Naturalism

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INDISPENSABILITY ARGUMENTS IN THE PHILOSOPHY OF MATHEMATICS have recently become the subject of much criticism (cf. Kitcher; Field; Sober). To my mind, the most compelling of these criticisms comes from Penelope Maddy in "Indispensability and Practice." Maddy considers two doctrines, owing primarily to W. V. Quine, which motivate indispensability arguments: confirmation holism and naturalism. Naturalism, in Maddy's view, is incompatible with confirmation holism; thus, indispensability arguments lose their support. In defense of indispensability theory, Mark Colyvan argues that Maddy's criticism depends on an unsatisfactory conception of naturalism. For Colyvan, under a proper (Quinean) understanding of naturalism, Maddy's incompatibility disappears.

So the debate turns on a conception of naturalism — each protagonist arguing for what they believe is the more "attractive" candidate. In my opinion neither account is suitable because both unnecessarily conflate realism with ontological commitment. Thus, Colyvan is correct to criticize Maddy's version of naturalism, but the Quinean naturalism he recommends does not provide a suitable alternative. Rather, a "compromise naturalism" that talks in terms of realism, not ontological commitment, provides the best defense of indispensability from Maddy's criticism. Since Maddy's criticisms attack the motivations for indispensability theory, we ought to consider these motivations in depth.

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Motivations for the Indispensability Argument

Due to Quine and Hilary Putnam, the doctrine of naturalism provides the primary support for the standard indispensability argument. Standard Quinean naturalism respects the methods of science, to the denial of first philosophy. Consider Quine's oft-quoted statement:

> Naturalism: abandonment of the goal of a first philosophy....The naturalistic philosopher begins his reasoning within the inherited world theory as a going concern. He tentatively believes all of it, but believes also that some unidentified portions are wrong. He tries to improve, clarify, and understand the system from within. (*Theories* 72)

Clearly, we are meant to use the methodology and theories of natural science as a starting point for philosophy. In particular, when we ask "what exists," we should first look to natural science for the answer. For Quine, the "ontological questions, under this [naturalized] view, are on a par with questions of natural science" ("Dogmas" 45). So under standard Quinean naturalism, we answer questions of ontology by consulting natural science's best theory of the world.

In this way, naturalism counsels us against an ontological commitment to entities outside of natural science—ghosts, say. To have ontological commitment to ghosts would be an affront to naturalism because it would disagree with science on nonscientific grounds. Note, however, that naturalism does not counsel us to incorporate all the entities of science into our ontology. Indeed, even scientists may view certain elements of a physical theory as merely useful fictions that help describe and predict observable results. The massless neutrino of particle physics might have fallen into this category at one time. Naturalism alone limits our ontology to the entities of natural science but does not require our ontology to contain all elements of natural science.

Complementing naturalism, confirmational holism is the thesis that scientific theories are confirmed and disconfirmed as wholes. The traditional notion of empirical confirmation, rigorously defended by Rudolf Carnap, holds that scientific statements are confirmed or disconfirmed by empirical evidence. For example, the evidence that confirms "F = ma" does not necessarily confirm other laws of Newtonian physics except

through a coincidence. Quine disagrees: "In taking the statement as [the] unit [of confirmation] we have drawn our grid too finely. The unit of empirical significance is the whole of science" ("Dogmas" 42). If a statement *s* in our best world theory is refuted by empirical evidence, then our whole world theory requires revision, not just *s*. Famously, Quine writes, "Our statements about the external world face the tribunal of sense experience not individually but only as a corporate body" (41).

The combination of naturalism and confirmational holism recommends an ontology containing *all* and *only* the objects of the natural sciences. The argument is straightforward. Let *S* be the set of entities from the language of natural science. Let *N* be a subset of *S* to which naturalism alone requires ontological commitment. Let M be the set *S*–*N*. By confirmational holism, observations that confirm theories using elements of *N* also confirm theories using elements of M. If theories involving *N* and M have the same support, then we would be holding an "ontological double standard" to give elements of *N* special ontological status over elements of M. Thus, we ought to have ontological commitment to all and only elements of *S*.

Unfortunately, many rival physical theories are confirmed by empirical results—a motivation for the indispensability requirement. Empirical data that confirms a physical theory of the world *T* also confirms a less attractive theory of the world *T'*. Indeed, *T* and *T'* may contain different entities altogether. This insight motivates the requirement of indispensability: we should have ontological commitment to all and only those entities that are indispensable to our best theory of the world. To paraphrase Colyvan, an entity is indispensable if either (1) it is not eliminable from our theory or (2) its elimination from our theory creates a less "attractive"[1] theory ("Arguments").

In all, considerations of naturalism, confirmational holism, and indispensability lead us to this conclusion: we should have ontological commitment to all and only those entities that are indispensable to our best theory of the world. What does this mean for mathematics?

[1]Colyvan suggests, correctly I believe, that a definition of attractive should "appeal to the standard desiderata for good scientific theories: empirical success; unificatory power; simplicity; explanatory power; fertility and so on" ("Arguments").

Mathematics and Indispensability

Much like quarks, leptons, and protons, entities of mathematics are indispensable to our best theory of the world. In fact, Putnam goes further, arguing that mathematics is the language that makes much of natural science possible. Measurement, for example, requires that we quantify not only things like distance and masses, but also quantify "over *functions from* masses, distances, etc. *to* real numbers, or at any rate to rational numbers" (74). For indispensability theorists like Putnam and Quine, mathematics is indispensable to our best theory of the world; thus, we should have ontological commitment to mathematical entities.[2] However, this conclusion brings the question, should we have ontological commitment to all mathematical entities?

Quine and Putnam argue that some mathematics is dispensable to science and does not warrant ontological commitment. The Quinean account suggests a tripartite view of mathematics. First, some entities of mathematics, like the rational numbers, are clearly indispensable to science because they are part of many successful direct applications of mathematics to science. Second, some entities of mathematics are indispensable to science because they form "simplificatory rounding out" of applied mathematics. These systems are not directly interpretable in the physical world but are formed in partial analogy to systems that are interpretable. Finally, some entities of mathematics are neither applied nor part of any reasonable simplification of interpreted mathematics. Predictably, Quine argues that mathematics in the first two categories require ontological commitment. That in the third category, however, Quine looks "upon only as mathematical recreation and without ontological rights" ("Reply" 400).

Having outlined the standard Quine/Putnam indispensability argument, we can begin to assess Maddy's criticism.

Maddy's Criticism

Maddy observes that the scientific community does not accept the existence of an entity simply because it is indispensable to a well-confirmed

[2]Field challenges Putnam on this point, arguing that mathematics is dispensable for science. He takes this dispensability to recommend a nominalist approach.

theory. Consider the case of atomic theory during the latter half of the nineteenth century.[3] Some physicists were skeptical of the atom's existence long after atomic theory had seen substantial experimental confirmation. These scientists, including Poincaré, certainly considered the atom a useful entity for explaining observable results but were unconvinced that the atom physically existed. For Maddy, historical considerations like these show that in the actual practice of science, scientists do not always equate empirical confirmation of an indispensable entity with physical existence.

For Maddy, this observation alone reveals a deep incompatibility between naturalism and confirmational holism. If we take naturalism seriously, we should respect the scientists' judgment and draw a "distinction...between parts of a theory that are true and parts that are merely useful" ("Indispensability" 281). From those entities of science that are "merely useful," we withhold ontological rights. However, confirmational holism requires us to grant ontological rights to *all* elements of our best scientific theory—even elements that are merely useful for fear of an "ontological double standard." Naturalism and confirmational holism differ on this point; thus they are incompatible.

Strictly speaking, Maddy's objection does not show that Quinean naturalism is incompatible with confirmation holism. Sure, Quine's naturalism limits our ontology to entities of our world theory, but it "believes also that some unidentified portions [of that theory] are wrong" (*Theories* 72). So if confirmational holism is correct, the turn-of-the-century skeptics were simply wrong. No incompatibility exists. If that is the case, then what does Maddy's objection purport to show?

To be precise, Maddy is arguing (1) that Quinean naturalism is too "weak" and (2) that the naturalism we ought to have is incompatible with confirmational holism. Maddy even admits, "Logically speaking, [the Quinean] holistic doctrine is unassailable" ("Indispensability" 280). However, Maddy wishes to convince us that Quinean naturalism diverges from scientific practice in ways that are intolerable for a "serious" naturalist. What's more, through this objection Maddy begins to spell out what kind of naturalism we *ought* to have. Maddy is clearly committed to a naturalism in which "if philosophy conflicts with [scientific] practice, it

[3]This is the example considered by Maddy ("Indispensability" 280–81).

is the philosophy that must give" ("Naturalizing" 176). Thus, right from the start this serious naturalism, which I will hereafter call "Maddy's naturalism," is incompatible with confirmational holism. We have seen one way in which Maddy's naturalism differs from Quine's. As her critique progresses, the two accounts diverge even more.

On Maddy's view, considerations of *mathematical* practice provide further impetus for abandoning Quinean indispensability theory. For example, continuum mathematics is arguably a paradigmatic case of mathematics' indispensable role in scientific theories. Thus, "the [indispensability] argument goes, we have good reason to believe in the entities of continuum mathematics, for example, the real numbers" (Maddy, "Indispensability" 284). The indispensability theorist, then, must accept that statements about real numbers and sets of real numbers have determinate truth-values—there must be a fact of the matter to the statement's truth or falsity. Maddy calls this *fact*. Now consider the statement *s*: Σ^1_2 are Lebesgue measurable. Because *s* is independent of the Zermelo-Fraenkel axioms, we cannot settle the "fact of the matter" by provability (Maddy, "Indispensability" 283). Rather, the mathematician relies on other, extrinsic, sources of evidence—aesthetics, new methods, intertheoretic connections—to determine this "objective" fact. In all, as long as real numbers are indispensable to our world theory, the truth of *s* is a legitimate mathematical question.

However, should real numbers become dispensable to our world theory, *s* ceases to have a determinate truth-value. The dispensability of real numbers to natural science would require the indispensability theorist to deny ontological rights to real numbers. Thus, questions about sets of real numbers, like the question of Lebesgue measurability, lose their meaning. Maddy states that "if all the remaining applications of continuum mathematics were explicitly understood as 'approximations' or 'idealizations' [then the] indispensability theorist would retreat to some version of [nominalism]," in particular with regards to *s* ("Indispensability" 286). Maddy calls this *no-fact*.

The first important point for Maddy is that the choice between *no-fact* and *fact* hinges on developments in physics. The problem of quantum gravity leads many physicists to suspect that space-time may not be continuous. If space-time is not continuous, it may be possible to remove continuum mathematics from all physical science. Clearly, "adherence to *fact* could be overthrown by progress in physics" ("Indispensability" 286).

The second important point is that this switch from *fact* to *no-fact* has methodological consequences that contradict mathematical practice. *Fact* recommends mathematicians use all their best methods to determine the correct truth-value of *s*. However, if developments in physics recommend *no-fact*, then mathematical methods must change. At worst, we consider the question of Lebesgue measurement meaningless and abandon the pursuit of a truth-value for *s* altogether—a clear change of methodology.[4] At best, the mathematician continues to investigate *s* holding "that there is no fact of the matter about [*s*], but that there are still good mathematical reasons...to adopt a theory that decides our question one way rather than another" ("Indispensability" 287). But this pursuit is based upon "mathematical canons of correctness" and not truth—another change of methodology.[5] In any case, developments in physics change mathematical methodology.

These two points constitute Maddy's *mathematical* practice objection to indispensability theory. Mathematicians do not, in practice, allow developments in physics to affect their methodology. In certain cases, however, indispensability theory recommends mathematicians adjust methodology in response to developments in physics. Thus, if we are "serious" naturalists, we should reject indispensability theory.

Here Maddy's naturalism departs from Quinean naturalism in a second way. Remember that Maddy first diverged from Quinean naturalism by asserting that science is privileged over philosophy, while Quine only asserts philosophy is *not* privileged over science. In her latest objection, Maddy departs further by extending her naturalism to cover mathematical practice. Quinean naturalism applies to only scientific practice.

Indeed, like Maddy's first objection, the mathematical practice objection is ineffectual using Quinean naturalism. Under the Quinean view, the move to *no-fact* still requires methodological change for the mathematician. This change reflects a move from mathematics with ontological rights to "recreational" mathematics. However, this methodological change does not conflict with Quinean naturalism because Quinean naturalism applies to only scientific methodology. For Quine, indispensability theory can and ought to recommend changes to mathematical

[4]Maddy calls this position *end of the story no-fact* ("Indispensability" 287).

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practice. Thus, the mathematical practice objection does not, strictly speaking, refute Quinean indispensability theory, it merely recommends Maddy's naturalism, which is incompatible with indispensability theory, over Quine's.

In all, we have a choice between two brands of naturalism: Maddy's and Quine's. Quine's naturalism has much to recommend it. First, it is compatible with indispensability theory, justifying an ontology of certain parts of mathematics. Second, it avoids troubling ontological "double standards" through its compatibility with confirmational holism. Quinean naturalism, though, does have the unfortunate habit of recommending change to both mathematical and scientific methods on philosophical grounds. On the other hand, Maddy's naturalism resonates with actual mathematical practice in ways that Quine's does not, but at the cost of abandoning indispensability theory. What's more, because Maddy's naturalism (1) puts practice above philosophy and (2) applies to mathematics, Maddy's naturalism commits us to rubber-stamping mathematical practice in troubling ways. For example, should mathematicians approach a certain area of mathematics using correct mathematical methodology, Maddy's naturalism counsels us to interpret the area in question realistically. It seems, then, that the act of mathematical investigation implies the existence of some class of mathematical entities (Colyvan, "Defence" 55–56).

Instead of choosing between the two brands of naturalism, I believe there is room for a compromise between them. Foremost, this "compromise naturalism" recognizes the distinction between realism and ontological commitment.

Compromise Naturalism

A realist believes (1) that the sentences of a given theory are true or false and (2) that something external to the human intellect confers this truth-value (Putnam 69–70). Notice that neither (1) nor (2) requires ontological commitment to entities of that theory. Rather, ontological commitment just provides one account of what is to be "external to the human intellect." There are other accounts—modal-realist accounts in particular—that also satisfy (2). While ontological commitment requires realism, realism does not require ontological commitment.

Compromise naturalism is defined to build on this distinction. Compromise naturalism has three properties:

Property 1: Like Quinean naturalism, compromise naturalism holds that we should have ontological commitment to only (not necessarily all) those entities of our best natural scientific theories.

Property 2: Like Maddy's naturalism, compromise naturalism is extended to mathematics.

Property 3: When investigators (from mathematics or science) employ sound methodology to determine the truth-value of a statement, compromise naturalism requires that we be realists with respect to that statement.

Having defined compromise naturalism, we can see how it avoids the pitfalls of both the Quine and Maddy versions of naturalism.

First, compromise naturalism "works" in indispensability theory. The first property of compromise naturalism limits our ontology to *only* entities of the natural sciences. By the "double standard" argument, confirmational holism expands our ontology to *all* entities of the natural sciences. Therefore, we should have ontological commitment to all and only those entities that are indispensable to our best theory of the world. This is exactly the same conclusion we reach using Quinean naturalism, so "swapping" compromise for Quinean naturalism does not affect indispensability theory.

Second, compromise naturalism is compatible with confirmational holism in both the situations outlined in Maddy's objections. Consider Maddy's first objection, where scientists were skeptical of the atom's existence despite its empirical confirmation in our best theory of the world. Indispensability theory argues that scientists were simply wrong to doubt the existence of the atom. Does this conflict with compromise naturalism? Compromise naturalism does not privilege science over philosophy, so indispensability theory's critique of atom skeptics is compatible with compromise naturalism in this respect. Only the third property of compromise naturalism has the potential to conflict with indispensability theory in this scenario. Fortunately, it does not. Since skeptical scientists came to their conclusion using sound scientific methods, compromise naturalism requires that we take a realist stance towards the atom's existence. That these scientists were wrong does not prohibit a realist stance. Thus, compromise naturalism is compatible with indispensability theory in Maddy's first scenario.

Compromise naturalism is compatible with indispensability theory in Maddy's second scenario as well. In this scenario, developments in physics coupled with indispensability theory have removed ontological rights from the real numbers. Presumably, mathematicians were pursuing the question of Lebesgue measurability using sound methodology. Thus, compromise naturalism requires us to be realists towards this question. Does the removal of ontological rights preclude us from being realists about the question of Lebesgue measurability? No. Ontological commitment requires realism, but realism does not require ontological commitment. Thus, compromise naturalism is compatible with indispensability theory in Maddy's second scenario.

Third, compromise naturalism does not advocate changes to the practice of mathematics. Removing ontological rights from the real numbers merely moves real-analysis from the domain of mathematics with ontological commitment to that of realist (non-recreational) mathematics. This switch does *not* have implications for mathematical practice. In both domains, there is an external fact of the matter concerning the truth-value of mathematical statements. The mathematician does not care why there is a fact of the matter, so methods do not change when her work moves from the domain with ontological rights to the realist domain.

To sum up, compromise naturalism avoids many of the pitfalls of Quinean naturalism and Maddy's naturalism. Namely, it preserves the consistency of indispensability theory without changes to mathematical practice. In this way, I believe compromise naturalism is a more attractive candidate than either Maddy's naturalism or the Quinean naturalism that Colyvan advocates. Before closing, I want to consider two possible objections to this conclusion.

Objections

First, one might object that compromise naturalism and indispensability theory advocate changes in scientific practice on nonscientific grounds.[6] In Maddy's first scenario, indispensability theory argues that

[6]Note that coupling compromise naturalism with indispensability theory does *not* advocate changes to mathematical practice on nonmathematical grounds. For example, the move from Platonist to realist mathematics did not affect methodology.

scientists were wrong to be skeptical of the atom— even if that skepticism was based on sound scientific methodology. While this does not show any inconsistency between compromise naturalism and indispensability theory, it does show how indispensability theory can refute scientific conclusions on nonscientific grounds. For those with "serious" naturalist tendencies, this seems to pose an intractable problem for compromise naturalism.

In response, I would argue that a good philosophy of science keeps scientists honest in exactly this way. I do not deny that in certain situations compromise naturalism coupled with indispensability theory can advocate changes to scientific practice. However, this does not automatically degenerate compromise naturalism. In Maddy's scenario, indispensability theory exposed the ontological double standard of skeptical physicists. Just because physicists followed accepted scientific methodology does not excuse them from unjustifiably (Quine would argue) denying ontological rights. Indispensability protects scientists from this intellectual "dishonesty." Indeed, where compromise naturalism and indispensability theory advocate changes to scientific method, I believe science benefits.

A second objection might be that it is hard to see how one can be a realist with respect to mathematical entities that do not exist. In the scenario where space-time is discrete, we admit that there is a fact of the matter about Lebesgue measurability of Σ^1_2 sets, but that these sets do not exist (i.e., they do not have ontological rights). What justification, then, do we have for saying there is a "fact of the matter" about these sets? Indeed, how can there be a fact of the matter about anything that does not exist?

In response, this objection only vitiates compromise naturalism if Platonism is the only form of realism. Fortunately, there are many other forms of realism. At its core, realism in mathematics is the thesis that mathematics makes assertions that are objectively true independent of our thought. To be sure, realism in mathematics has traditionally been associated with object Platonism, but Platonism is not the only form of realism. Different variants of structuralism and modalism, for example, have worn the realist mantle (cf. Resnik 239–40; Hellman). As long as there exist alternate accounts of realism, compromise naturalism can assert there is a fact of the matter about nonexistent entities.

For Maddy, considerations of practice justify our rejecting not just Quinean naturalism, but indispensability theory altogether. Compromise

naturalism allows us to preserve indispensability theory without the changes in scientific and mathematical practice Maddy finds objectionable Considerations of practice may force us to abandon Quinean naturalism, but they do not invalidate indispensability theory

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