# A Russellian Critique of Kurt Gödel's Ontological Proof

## NOAH VALDEZ

his paper presents Austrian mathematician Kurt Gödel's famous Ontological Proof of God's existence against the writings of British philosopher Bertrand Russell, and, in doing so, attempts to imagine a Gödelian response to any possible criticisms from such works as The Philosophy of Logical Atomism (1918), Is There An Absolute Good? (1922) and On Denoting (1905). Gödel's proof represents one of the first so-called 'modal ontological proofs' of God, and, since its initial 1970 publication, has catalyzed numerous philosophical and theological debates over its general axiomatic structure and logical description of divinity. Nevertheless, in spite of the depth and diversity that usually pervades these dialogues, seldom, if ever, does the role of language within the proof occupy a position of primary importance. If Gödel is to use such words as "God" or such phrases as "a God-like being," then it is essential that we understand the meaning of these terms and challenge, if need be, Gödel's usage of them before accepting the proof prima facie. What follows are two distinctive critiques that each problematize some aspect or inadequacy within the proof's assorted axioms, theorems or definitions. The first critique questions the legitimacy of the "absolute good" within Gödel's

Noah Valdez is a junior at Boston College majoring in philosophy and minoring in English. He will apply to graduate schools in philosophy and comparative literature this winter with the intention of pursuing academia one day. His philosophical interests are absurdism, early and existential phenomenology, philosophy of language, Derrida, and philosophy of literature. provision of positive properties, and whether or not such properties name an objective system of morality or merely fall susceptible to grammatical fictions. The second critique challenges the method by which Gödel defines God, and draws upon Russell's theory of descriptions as an objection to Gödel's general manner of naming. The enduring hope of this paper remains an attempt at developing a more comprehensive understanding and sense of both Gödel's and Russell's theories, or at the very least, to translate both of them into a semi-compatible grammar.

# 1. A Description of the Ontological Proof (1970)

Prior to any comparative or critical review of Gödel's Ontological Proof, it is first necessary to outline the entirety of the 12-point argument so as to avoid ambiguity and to resolve any terminological/symbolic misunderstanding. I have chosen the original 1970 edition for a number of reasons,<sup>1</sup> effectively rejecting subsequent reformulations and revisions made by Dana Scott in the appendix to *Sobel 1987* (Benzmüller and Paleo 307–313). The proof is represented in the following figure:

Axiom 1:	Either a property or its negation is positive, but not both: $\forall \phi[P(\neg \phi) \equiv \neg P(\phi)]$
Axiom 2:	A property necessarily implied by a positive property is positive: $\forall \phi \forall \psi [(P(\phi) \land \forall x [\phi(x) \supset \psi(x)]) \supset P(\psi)]$
Theorem 1	: Positive properties are possibly exemplified: ∀φ[P(φ) ⊃ ◊∃xφ(x)]
Definition 1	: A God-like being possesses all positive properties: $G(\mathbf{x}) \equiv \forall \boldsymbol{\varphi}[\mathbf{P}(\boldsymbol{\varphi}) \supset \boldsymbol{\varphi}(\mathbf{x})]$
Axiom 3:	The property of being God-like is positive: P(G)
Corollary:	Possibly, God exists: <b>\$\deltaxG(x)</b>
Axiom 4:	Positive properties are necessarily positive: $\forall \phi[P(\phi) \supset P(\phi)]$
<b>Definition</b> 2: An essence of an individual is a property	

Definition 2: An essence of an individual is a property possessed by it and necessarily implying

<sup>&</sup>lt;sup>1</sup> One such reason is Dana Scott's rejection of necessitarianism in his revision of Gödel's proof, which, inevitably, implicates the proof's definition of 'essence' and creates contradictions in Gödel's metaphysics.

- **Theorem 2**: Being God-like is an essence of any God-like being:  $\forall x[G(x) \supset G \text{ ess. } x]$
- **Definition 3**: Necessary existence of an individual is the necessary exemplification of all its essences:  $NE(x) \equiv \forall \phi [\phi \text{ ess. } x \supset \Box \exists y \phi (y)]$
- Axiom 5: Necessary existence is a positive property: **P(NE)**

Theorem 3: Necessarily, God exists: DaxG(x).

(Benzmüller and Paleo 307-313)

Although his philosophical journals (specifically "Phil XIV") indicate that Gödel created this proof sometime between 1946-1954, he hesitated to publish it until 1970 possibly "for fear it would be thought 'that he actually believ[ed] in God'" (Adams 388).<sup>2</sup> As a result, Charles Hartshorne and Alvin Plantinga receive much of the credit for being the first to introduce modal ontological proofs of God, but there exists little evidence to suggest that these works influenced Gödel in any philosophically significant way (Adams 391).<sup>3</sup> Nevertheless, our purposes are strictly evaluative, not biographical, and as such, there are two potential areas of ambiguity that require further clarification in the above-stated proof. The first concern emerges around the meaning of positive and negative properties in Axioms 1 & 2, but more importantly, whether these properties are as mutually exclusive in Gödel's metaphysics as in traditional mathematics (e.g.  $\neg P \lor P$ , per the 'law of non-contradiction'). His answer to the initial question is seemingly simple, for he suggests a "more Leibnizian interpretation of positiveness... [as] pure 'attribution' as opposed to 'privation' (or containing privation)" (Adams 397).<sup>4</sup> This definition of course receives its philosophical parentage from Leibniz's That an Ens PerfectissimumExists (1676), which envisions God to possess solely positive, or perfective qualities in the "moral aesthetic sense" (Hazen 364). It thus follows that negative properties must come to

 $<sup>^2\,</sup>$  Or at least, this remains the opinion of Robert Merrihew Adams. It certainly conflicts with Gödel's quite public alignment with theism.

<sup>&</sup>lt;sup>3</sup> Hartshorne's proof resembles something more typical of Anselm than Leibniz, however. It is included later in this article; see Section 4: *The Descriptivist Critique*.

<sup>&</sup>lt;sup>4</sup> Pure attribution functions much like 'substance' in Spinoza's *The Ethics*, of which Leibniz had read and consulted its author on extensively. I present the definition of substance here, which is "what is in itself and is conceived through itself, that is, that whose concept does not require the concept of another thing."

represent restrictions and/or imperfective qualities within this context, for God is a type of maximum of perfection that precludes the possibility of limitation (even if this last claim should appear to be one such example).

Furthermore, in order to meet the requirements of Definition 1, positive qualities must also be conjunctive and non-contradictory with each other, meaning that any possible composition of positive qualities in a given system G(x) is compatible with itself. Both Gödel and Leibniz agree on this last point since incompatibility can only be derived by combining two contradictory conjuncts together (e.g. 'thou shalt not murder' and 'thou ought to murder'), yet certain thinkers like Descartes remain suspicious for fear of those instances where positive properties might exclude each other "without...being formal negation[s]" (Adams 394).<sup>5</sup> Lastly, positive properties are always and necessarily positive since even the predisposition toward negation would indicate the presence of inferiority (by means of degradation). Hence, Gödel's God must follow the moral heuristic that "the greatest advantage + the smallest disadvantage is negative" in order for P( $\varphi$ ) to stay positive in Definition 1 (Adams 388).

The second major concern regards the question of essence, which, for Gödel's purposes, can be defined as the property of an individual that entails all of its other properties (i.e.  $\Psi(x) \supset \forall y(\varphi(y) \supset \Psi(y))$  where "y" represents the totality of properties implicated by an essence " $\psi$ "). What concerns us here is not so much the ambiguity of Definition 2 as much as the trend towards interpreting essence as only those features most fundamental to the constitution of an object. As Saul Kripke likes to demonstrate, "Important properties of an object need not be essential," and in fact, can be contingent (85). For example, Kurt Gödel was a professor at Princeton, but he could have taught elsewhere, and in effect, assumed a different character (e.g. a professor at Harvard) whilst still maintaining the same essence (Hazen 365). Therefore, Kripke says, when we speak of the essence  $\Psi(x)$  of any object, we must also comment on that object's character in order to capture the entirety of properties associated with it (since character is of a higher logical order). Kripke's supplement to Definition 2 appears reasonable enough, but inevitably, proves to be superfluous since both Leibniz and Gödel understand all truths as necessary truths (Hazen 369). That is to say, "whatever happens, happens for a reason," and thus makes the classification of characteristics as contingent theologically incoherent since all properties are essential (Hazen 369).

<sup>&</sup>lt;sup>5</sup> Yet, he never gives an example of what one such conjunct might look like.

Another common objection to Gödel's description of essence (specifically Theorem 2) involves whether or not he unintentionally proves the existence of a Devil-like being D(x), insofar as the inverse of Theorem 1 must also be true (i.e. negative properties are also possibly exemplified,  $\forall \phi[\neg P(\phi) \supset \Diamond \exists x \phi(x)]$ ). This particular argument for the existence of a Devil-like being, however, is rather weak, for it overlooks Axiom 5 that necessary existence is a positive property, not a negative one, and consequently excludes the Devil-like being's possession of it. The more threatening objection over the existence of a Devil-like being comes from Petr Hájek, who states that the "disjunctive property of 'being God-like or being Devil-like,' say ( $\lambda x.Gx \vee Dx$ ), is positive...since this property is a consequence of a property of 'being God-like' that is itself positive" (Kovač 581). Gödel's God possesses infinite possibility, including the possibility to choose between being God-like or being Devil-like, because, while being Devil-like is not a positive property, choosing not to be Devil-like is. It only logically follows that this choice of either/or would be positive (and thus true), for as Gödel demonstrates in Axiom 2, any property implied by a positive property is positive. The danger in all of this is that God under Gödel's schema possesses the property of potentially choosing to be Devillike, which, naturally, is a concerning problem for an entity supposed to be immutably good and moral. One can imagine the mathematician being very particular about the distinction between being and choosing to be, for although God can always choose to be Devil-like, he never does.

# 2. The Critique of the Absolute Good

The first criticism against Gödel's proof comes from a short text titled *Is There An Absolute Good*?, which Russell prepared in the Spring of 1922 for a presentation to 'The Society' at Cambridge (148). Its most immediate purpose lied in the interrogation of G.E. Moore's conception of the 'absolute good' per *The Principia Ethica* (1903), and inevitably, hoped to demonstrate the untenability of such a belief due to Russell's theory of 'incomplete symbols' in *The Philosophy of Logical Atomism*. I will first explain the thesis behind Russell's evaluation of Moore's morality before proceeding to relate its applicability to Axioms 3, 4 & 5 of Gödel's proof. For Russell, the problem with using terms like "the good" in sentences such as "M is good," arises not so much from logic as much as from grammar (Russell,

Is There An Absolute Good? 148).<sup>6</sup> The "good" in that sentence functions not as a predicate but as an incomplete symbol, which "are things that have absolutely no meaning whatsoever in isolation but merely acquire a meaning in context" like, for example, "Scott is the author of Waverley" (Russell, The Philosophy of Logical Atomism 91). That is to say, whereas "Scott" might exist as a name that can be pointed to (or towards), "the author of Waverley" is neither a name nor a constituent part of the proposition, and thus, cannot exist in the same way that "Scott" does. Therefore, Russell explains, Moore's ethics falls into the fallacy of attributing existence to the "absolute good" in the same manner that one mistakenly grants existence to "the author of Waverley." Moore explains that when we use "the good" in language, what "we mean by it [is] that quality which we assert to belong to a thing," such that we associate "emotions of approval" with A, B, C and emotions of disapproval with X, Y, Z (9). The drawback to this account of the good is that the designation of something with approval or disapproval only enters into consideration at its initial classification, for nothing binds together A, B, C other than the fact that we approve of them. "A" is good because it possesses the same qualities as "B" and "C," but "B" is good because it possesses the same qualities as "A" and "C." Thus, our definition becomes recursive ad infinitum (Russell, Is there an Absolute Good? 149).

At the risk of taking the connection between Moore and Gödel to be intuitively apparent, I will describe its relationality here, for many of Gödel's axioms supposedly fall into the same fallacy of incomplete symbolism. In Axiom 3, "the property of being God-like is positive," which admittedly is not a statement that Russell would oppose unless positivity is defined in "the moral-aesthetic sense," as Gödel proceeds to do (Hazen 364). The same complaint is equally true of Axiom 4, that "positive properties are necessarily positive," and Axiom 5, that "necessary existence is a positive property," for in both of these instances, Gödel amounts to saying that positive properties are necessarily good/perfectible and necessary existence is a good/perfectible property. There are three imaginable answers to this, the first of which advanced by Moore on behalf of other likeminded moral realists. Russell attempts to make a distinction between "non-existent properties," or predicates that cannot exist in isolation but only in propositions (viz. the good), and "non-natural properties," or predicates that cannot exist both in isolation or in propositions (or at least, meaningfully that is) (Russell, The Philosophy of Logical Atomism 90). The

<sup>&</sup>lt;sup>6</sup> For those familiar with the incomplete symbolism behind such statements as "The current King of France is bald," the same is true here except in the proposition's predicate, not its subject.

problem with this view is that if the good is an example of one of these non-existent properties, then it must also be a non-natural property, for how could we come to be acquainted with (and use in language) something that can exist but cannot be defined? As Moore claims, "far too many philosophers have thought that when they named those other properties they were actually defining the good; that these properties...[were] entirely the same with goodness. This view I propose to call the naturalistic fallacy" (Moore 10). So it seems that Russell would need to give up his fundamental theory of acquaintances in order for his critique against Moore's 'absolute good' to succeed, since we can use and understand phrases like 'the good' in propositions without knowing their definitions *per se*. Furthermore, one can imagine Gödel contesting Russell's claim that the good does not exist within his proof, for in the statement "God is good," the good exists as a positive property contained within the system G(x).

The second possible answer against Russell's criticism relies upon his understanding of 'disjunctive facts,' such that whenever one utters "p or q," he or she must find two corresponding facts in order to prove the truth or falsehood of the proposition (Russell, The Philosophy of Logical Atomism 39). What Russell overlooks is that while A, B, C are connected insofar as they gain approval, and X, Y, Z insofar as they do not, both sets at the very least also possess the property of being "A, B, C" and "X, Y, Z" respectively.7 Admitting to the truth of that last statement not only demonstrates an instance of disjunctive properties in Russell's own counterexample (something imaginably disconcerting to his metaphysics), but also hints at the feasibility of such properties existing logically in Gödel's proof. Perhaps the most famous allegedly disjunctive property is his statement of choosing to be God-like or Devil-like at the same time (as discussed previously), for this appears to be an example of "p or q" that stems from only one fact in the world, that is, "being God-like." The point to consider in all of this is that disjunctive facts are not as easily dismissible as other logical fallacies in language, and that Gödel's proof hints at one reason that we ought to be more precautious. Using Russell's own hesitancy against arriving at general truth-statements by means of induction, we ought to not be so quick as to say that "There are no disjunctive facts," unless we have "enumerated all facts" (The Philosophy of Logical Atomism 70). Yet, the last part of that quote proves to be an impossibly long task, and for now, the question of disjunction must remain unresolved.

<sup>&</sup>lt;sup>7</sup> This at least would appear to be one instance of an identifiable disjunctive property.

The last counterargument against Russell's objection critiques his statement "that [if] our ethical judgements claim objectivity," then they are "all false" (Moore 9). For Gödel, it is not humanity that determines ethicality within the proof, but instead, a God-like being. While the proof makes an argument for the existence of God as constructed in a certain way, it does not extend that claim to humanity. In this way, if God exists alone, then his judgements necessarily constitute the entirety of morality since they compromise the only emotions of approval or disapproval in existence. But, if humanity does exist and God alongside them, then traditional theology demands that his omnipotence supersede their ethical sentiments. Thus, Gödel not only can prove the existence of the good per Moore's *Principia Ethica*, but also that this good is objective and that it must come from God.

#### 3. The Descriptivist Critique

The second critique problematizes Gödel's usage of names in Definition 1, specifically, the name of God. As Russell would contend, there exists no rationale that the system G(x) must be a sentient entity (i.e. a God-like being) instead of a logical class or series (i.e. of which all positive properties would be merely a part). Before investigating this claim further, we would do well to clarify what Russell means by "names" and their distinction from "descriptions." As he states in the Philosophy of Logical Atomism, "[a] name, in the narrow logical sense of a word whose meaning is a particular, can only be applied to a particular with which the speaker is acquainted," and thereby excludes the naming of anything with which the speaker is unacquainted (9). What we conventionally think of as names, say "Socrates," are in fact abbreviations for a whole class or series of descriptions, such as "The teacher of Plato," or "The philosopher who drank the hemlock" (Russell, The Philosophy of Logical Atomism 9). The only words that we use as names in the proper logical sense are demonstrative pronouns like "this," or "that," so when Gödel says that a "God-like being possesses all positive properties," he is actually describing the class G(x)as God, not that there is a *particular* named God. This constitutes the first problem for Gödel's proof, insofar as he takes an inductive leap in assuming the presence of the class to indicate the presence of a particular, when in reality they are two separate and distinct things. The second problem for the proof comes from Russell's On Denoting (1905), which is best described in a footnote to pg. 491:

The argument can be made to prove validly that all members of the class of most perfect Beings exist; it

can also be proved formally that this class cannot have more than one member; but, taking the definition of perfection as possession of all positive properties, it can be proved almost equally formally that the class does not have even one member.

(Russell, On Denoting 491).

Thus, Gödel's problems appear to be compounded, for by the preceding criteria, he must both prove that any value of "x" meets the restrictions of a class G(x), and also that this class G(x) can exist as a particular in the same way as a "this" or a "that" (i.e. as a name). The third problem is as much a concern for Russell as it is for Gödel, and demonstrates a fundamental flaw in the descriptivist methodology of denotation. In Naming and Necessity, Kripke asks us to imagine that Gödel did not create the incompleteness of arithmetic proof, and that instead, he stole it from some other man named "Schmidt" (Deutsch 446). Keeping this in mind, we could easily identify the proposition "Gödel is the creator of the incompleteness of arithmetic proof" as false due to our knowledge of Schmidt. This time, however, imagine another scenario wherein we know nothing of this man named Schmidt, despite his being the creator of the proof. Is our previous proposition now false, for in the absence of any knowledge about Schmidt, it would seem that Gödel is the proof's rightful founder (Deutsch 446)? Russell in such scenarios thought that when we refer to individuals like Gödel we are in fact referring to Schmidt, but if this is true, then we certainly would not know it, and a strange sense of ambiguity would appear to pervade almost every proposition for fear of it being something else. Thus, when Gödel refers to a God-like being throughout the entirety of the proof, is he referring to God, or is he referring to (and consequently proving) some other entity unbeknownst to us?

I would like to begin with a response to the objection made in On Denoting with a phenomenon that William Van Orman Quine described in On What There Is (1948) as "Plato's Beard". If there are two contrasting ontological systems, one proposed by "McX" and another by "I," neither one of them can disprove the existence or non-being of the other without granting some level of ontological legitimacy (even if only through the act of criticism) (Quine 21). Assuming that Russell was able to prove that no value of "x" could fit our membership restrictions for G(x) because of Plato's Beard, he would still have to admit to the existence of G(x) as an unactualized possibility. In his citation of "x," he has imagined the possibility of conditions for membership to the class G(x), and thereby given a form of existence to the possibility insofar as it does not meet the

actuality of x. The inconsistency within this line of reasoning unveils itself when Russell admits God to be an unactualized potential—insofar as he is also admitting to the actuality of God per the following proof:

- (i) N  $[(\exists x)G(x) \supset N(\exists y)G(Y)]$  It is necessary that if God exists, God exists necessarily.
- (ii)  $M(\exists x)G(x) \supset MN(\exists y)G(y)$  If God possibly exists, then God possibly exists necessarily.
- (iii)  $M(\exists x)G(x) \supset N(\exists y)G(y)$ . If God possibly exists necessarily, then God necessarily exists because God exists necessarily (Adams 390).<sup>8</sup>

While this particular proof represents the authorship of Hartshorne– not Gödel–both agreed that "the logical underpinning of the argument is that *to be possible* and *to be actual* are the same thing" (Ernst 61). Thus, when Russell states that a value for the class G(x) possibly, but does not actually exist, he fails to recognize that in conceding to the possibility of a value of 'x' that meets G(x) he concedes in fact to the actuality of that value's existence for those three points listed above.

Moving on to the critique forward by The Philosophy of Logical Atomism, even if we admit to the possibility (and consequent necessity) of an "x" for G(x), how are we to know that this is an entity instead of some generic class or series (operating in much the same way that a category might)? This is a valid concern, but, alas, proves altogether inconsistent in light of Russell's position on existence-propositions. He says that "you know that there are people in Timbuctoo, but I doubt if any of you could give me an instance of one"-in effect attempting to prove that one can know an existence-proposition without knowing any actual values that would satisfy or fulfill it (Russell, The Philosophy of Logical Atomism 68). The problem with this assertion of course is that there are in fact people living in Timbuctoo, whether or not we are acquainted with them. In this way the class of "people living in Timbuctoo" is both a categorical description (*i.e.* for any people that find themselves to be living within Timbuctoo) as well as a factual one (i.e. for people currently living in Timbuctoo). We have already established G(x) to be a logical class, but in light of the Timbuctoo example, it remains possible for G(x) to be some entity that

<sup>&</sup>lt;sup>8</sup> This proof was published in 1962, but its true philosophical parentage comes from Anselm's Ontological Argument.

we are unfamiliar or unaware of. If Russell's hesitancy to admit of an "x" that fulfills G(x) stems solely from his unacquaintance with "x," then we must abandon his position in much the same way that we would abandon the categorization of "people living in Timbuctoo" as vacuous (for indeed, it is not). The last objection concerns whether or not when we talk about God we are referring to him and not some other entity that meets our aforementioned definitions. Kripke's writings are quite resourceful in this domain of inquiry, as he distinguishes between the semantic meaning of a proposition versus its pragmatic meaning. It is "roughly the distinction between the proposition(s) conventionally encoded by a sentence and the proposition(s) a speaker intends to communicate," such that when we say Gödel proved the incompleteness of arithmetic, who we mean to reference is Schmidt (in keeping with the proposition's pragmatic meaning) (Deutsch 455). As a result of this distinction, should our proof mistakenly refer to G(x) when we instead intend to refer to some other entity-hence G(x)-our pragmatic referent nevertheless remains true. The error then concerns our own semantical inability, which logically, would not render the proof false.

# 4. Conclusion

In bringing forth these two critiques against Gödel's Ontological Proof of God, it is important to remember that the original intention was never to convince the reader of either Gödel's or Russell's view, so much as it was to bring these two writers into conversation with each other. Admittedly, the arguments made here are far from exhaustive (even from within Russell's own canon of work), but alas, the hope is that a higher standard for precise syntax and the symbolic meaning contained therein has been achieved. Despite arguing on Gödel's behalf against Russell's varied objections, the truth is that much of the terminology that Gödel develops and refers to proves quite vague and ambiguous (perhaps even deliberately so). If we are to take Anselm's definition of theology as "faith seeking understanding," then we must also question Gödel's religious context and its consequences—if anv—to his usage of language (Williams). Robert Adams is correct that a small but growing amount of secondary source literature has begun to emerge around Gödel's proof, for it is by no means a trivial argument, and it is my ambition to have contributed to this dialogue in some minor, but not entirely insignificant manner.

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