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On the Infinite Divisibility and Composition of One-Dimensional Continua: An Ancient and Medieval Perspective

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SOME PEOPLE have, on a pensive occasion, considered the following question: In what sense do parts make up a whole? Generally this question arises in regard to everyday, three-dimensional objects: What are the fundamental parts which compose tables, doorknobs, or lawn furniture? For most, this philosophical quest ends when they arrive at the physical level of atoms and molecules. For the more scientifically inclined, the question may end at the level of highly elusive subatomic particles like electrons and muons. However, there is no reason to suppose that even these subatomic particles cannot be "split" as well, at least intuitively. If they are spatial objects, then how could they not have parts? If they do not take up space, how could they form normal, three-dimensional objects with magnitude? Perhaps they could be divided *ad infinitum*. These questions roughly constitute the problem of the continuum.[1]

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[1]The above example concerning normal objects illustrates the problem of the continuum in three dimensions. The problem in three dimensions concerns how two-dimensional planes form an object of three dimensions. Similarly, there is the problem at the level of two dimensions, namely how lines form planes. In this essay, however, I will deal with the one-dimensional form of the continuum problem: In what sense do points constitute lines?

I pick up the discussion concerning the nature of the continuum from the medievals. The following argument illustrates a typical position of that time.[2]

1. If something can be done, to suppose that it has been done involves no impossibility. (assumed)
2. If a continuum can be infinitely divided, it involves no impossibility to suppose it has been infinitely divided. (from 1)
3. But, if a continuum has been infinitely divided, then either (i) there actually exists an infinity of indivisible but nonetheless still extended parts (i.e., atoms); or (ii) there actually exists an infinity of unextended parts (i.e., indivisibles). (assumed)
4. It is impossible (i) that there be an indivisible and yet extended part, and it is impossible (ii) that a continuum be made up out of an infinity of unextended parts. (assumed)
5. So, it does involve an impossibility to suppose a continuum has been infinitely divided. (from 3, 4)
6. So, a continuum cannot be infinitely divided. (from 2, 5)
7. A continuum cannot be infinitely divisible. (from 6)

The medievals were directly responding to Aristotle. The concept that continua are infinitely divisible is essentially Aristotelian.[3] Because the above argument concludes by denying this concept, it is ultimately anti-Aristotelian. Divisibilism holds that there exist no such entities as indivisibles, whether they be indivisible parts with extent (e.g., atoms), or indivisible parts without extent (e.g., indivisibles). Roughly speaking,

[2]This argument is slightly adapted from an argument that Aristotle attributes originally to Democritus. See *De generatione et corruptione*, 316a14. It is formally paraphrased in Kretzmann's "Adam Wodeham's Anti-Aristotelian Anti-Atomism," 381–82. Kretzmann claims that Aristotle believes it was a variation of this argument that must have convinced Democritus of the existence of atoms.

[3]As opposed to 'division everywhere' which derives from Zeno's paradoxes and their utilization by the pre-Socratic atomists, e.g., Democritus. I will explain this distinction in the discussion of premise (4).

Aristotle argued for the divisibilist position.[4] Atomism is the position which states that continua are composed of a finite number of indivisible parts with magnitude—atoms. Indivisibilism, on the other hand, maintains that continua are composed of indivisible parts without magnitude—points, instants, etc. Divisibilism denies both positions. It holds that continua are composed of ever-divisible parts with magnitude—lines, stretches, or periods of time, etc. Although the above argument might seem like a negative argument for divisibilism by demonstrating, in premises (1) through (6), the implausibility of the other above-mentioned perspectives, it aims to show that continua cannot be infinitely divisible (premise 7).

It will be my contention that this argument fails. But despite its unsound outcome, the argument is useful. An analysis of each of the premises will serve as a stepping-stone to further explore this multifaceted debate concerning the composition and divisibility of continua. Premises (1) and (2), I conclude, are justified. I attack premise (3) on the grounds that there remains a further possibility not considered, i.e., an infinitely divided line may consist of no parts. I will then attack part (ii) of premise (4) on grounds similar to the objection to premise (3) that an infinitely divided continuum may consist of an infinite number of indivisibles. Moreover, I will raise an atomistic objection to part (i) of (4), but conclude that in the end it remains insufficient. While these objections would be adequate to override the entire argument, the argument has yet another place vulnerable to attack: it involves an invalid move from premise (6) to the conclusion in (7). I will finish by endorsing the Aristotelian position.

Premises (1) and (2)

Premises (1) and (2) may seem plausible. If doing x is a real possibility, then there seems to be no logical impossibility in supposing

[4]Actually, Aristotle might be understood better as a moderate divisibilist, as he held that the boundaries of continua, as opposed to their internal components, are indivisible. See Aristotle's *Categories* 4b22 and *Physics* 231a–b. Nonetheless, concerning the concept of infinite divisibility and the composition of continua, labeling him as a divisibilist does his thought no disservice.

that x has, in fact, already been done. For example, if it is possible that I could run a mile in under five minutes, then the supposition that I have already done so would not be logically impossible. Similarly, the instantiation of the particular terms found in (2), i.e., 'a continuum' and 'infinitely divided', for the universal terms found in (1) seems justifiable.[5]

Let us, however, take a closer look at premise (1). If we separate the antecedent from the consequent and analyze the respective verbs, we begin to see how the claim may not be plausible in all cases. The verb phrase in the antecedent, 'can be done', involves the modal auxiliary 'can', which does not refer to any tense. If x can be done, then x could have been done, could be being done, or could be done in the future. Yet the verb phrase in the consequent, 'x has been done', clearly refers to the past tense. So, the pressing question is whether (1) would remain true when the 'something' with which it is concerned could happen *only* in either the present or the future and not the past.

There may be some who claim, however, that there are sound objections which involve past facts. A possible objection of this kind might proceed as follows: Imagine one was, *in fact*, sitting at time t_1 (in the past). Even if it is possible that he can walk in general (he is a normal human being with two legs, etc.), to suppose him to be walking at t_1 is impossible. Thus, premise (1) is not true in all cases.

This objection fails, however. Should we be consistent, it outright denies the antecedent. It is not even possible that he can walk at t_1, since it is a *timeless fact* that he was then sitting. It seems to me that all such objections to premise (1) involve this inconsistency. As indicated above, we consider words like 'can', 'would', 'may', to be timeless. This is also true of facts—whether they be facts about the past, present, or future. Thus, if we have already posited a supposition concerning a timeless fact about the past, any subsequent objection that presupposes a denial of this fact cannot even be a possibility. Therefore, all objections of this nature fail.

Some may contend, however, that there are 'somethings' which can happen only in either the present or the future. For example, if the

[5]In spite of the fact that modal auxiliaries can often introduce fallacies when they are mixed with quantifiers, the move from (1) to (2) seems to me but a variant of the universal elimination rule, which states that from any universal proposition with the form $(x)Fx$, one can derive any particular, e.g., Fm.

universe can end, would the supposition that it has already ended involve an impossibility? Of course, this depends upon whether time ceases to exist when the universe does. If we accept this assumption that time only relationally exists, then premise (1), in this case, would be nonsensical. For the consequent involves a supposition which could be possible only from a future perspective. But if time ended, then there would be no future perspective by which to look back into the past. Thus, even such a supposition is an impossibility. So, by this example alone, we can see that premise (1) may not be universally true.

The above objection nonetheless rests upon a rather arguable assumption. I argue that there are no other possible candidates which could satisfy the requisite conditions for an adequate objection to (1). There are no other 'somethings' which could only be done in either the present or the future and not the past. Moreover, I believe all objections concerning timeless facts occurring in the past fail on the grounds of inconsistency. Thus, premise (1) remains plausible.

Let us now consider the putatively sound move to premise (2). Perhaps one might prematurely object here by claiming that an infinitely divided continuum could not possibly exist. The objector might argue that the process of infinite division would never end, so supposing a continuum to have been infinitely divided is contradictory. Whether or not a continuum can or cannot be infinitely divided is a significant question that will surface later in this essay.[6] The argument at this point, however, remains conditional. That is to say, *if* a continuum can be infinitely divided, then it involves no impossibility to suppose that it has in fact already been infinitely divided.[7]

[6]Especially when considering the move from premise (6) to (7).

[7]It is precisely on these grounds that I chose not to raise the objection to premise (1) concerning ongoing change, i.e., change which never ends. Take, for example, the futile project of determining the exact value of π. The fact that it seems an impossible endeavor is irrelevant to the claim as it exists in its conditional form. If it is true that one can determine the exact value of π, then premise (1) would encompass this situation. If it is false, then such considerations would lie outside the scope of premise (1). However, whether it can actually be done or not is irrelevant to our present purposes.

Premise (3)

Premise (3) concerns the ontology of an infinitely divided continuum. It presupposes that if a continuum has been infinitely divided, then an infinite number of parts must exist. Then it asserts that the only option for this infinite number of parts is that either (i) they are parts with magnitude like atoms, or (ii) they are parts without magnitude, like points or instants, i.e., indivisibles. This distinction is designed to cover all of the possibilities.

Perhaps, however, there is room to object to the assumption that if a continuum has been infinitely divided, then there necessarily would exist an infinite number of parts. Let us assume that the process of infinitely dividing a finite line entails halving the original line, then halving the remaining two lines, then halving the remaining four lines, and so on. Now let us suppose that this process has been sufficiently completed, and the line is thus infinitely divided. It is not clear that what remains would be an infinite number of parts either with or without magnitude.[8]

One could suggest that if a continuum were infinitely divided, then there would exist *no* remaining parts. A distinction between actuality and potentiality may make this better understood. Let us imagine a line being infinitely divided in the manner described above, except that after each division the remaining lines are *actually* separated. One would first "cut" the line in half, thereby leaving two equal halves separated from each other. When the line originally existed with the *potential* of being halved, these potential halves shared at least one part of the line. This shared part is exactly what made the original line one line, and not

[8]First of all, I must admit that it is an assumption that infinite division should occur in this fashion. It certainly accords with the principle nonetheless and is adequate for our present purposes. Secondly, one might suggest that it involves a logical impossibility to suppose that this process could be completed. This idea is perhaps implicitly lurking behind my subsequent objection to step (3). A possible reply might be to the extent that from the eternal perspective, God would be able to have completed the process. Nevertheless, since the argument has already supposed the process to have ended, it forces us to conceive what an infinitely divided continuum would look like.

any more than one line, in the first place. Upon actually halving the line into two separate and equal lines, either (A) the remaining lines continue to share the part, or (B) the remaining lines cease to share the part. (A) is impossible due to the fact that sharing the part was precisely the reason that the original line remained a coherent and unseparated whole. So, if two potential halves shared the part, they would not be separated. They are separated, so they do not share the part anymore.

Thus, we must conclude that (B) the remaining lines cease to share the part. If (B), then either (B1) one of the remaining lines has the part and the other does not, or (B2) both of the remaining lines have parts of the part, but neither has the whole part, or (B3) both of the remaining lines do not either have the whole part or even parts of the part. If (B1), then it would be impossible for the two remaining lines to have been halved equally, as one line would have one more part than the other. So, (B1) is not an option. If (B2), then we are correct to ask what part of the part was shared by the original line. Upon recognition of this part of the part, we are faced with the initial problem once removed. Either (A) the remaining lines continued to share the part of the part, or (B) the remaining lines ceased to share part of the part. Thus, (B2) seems to be embarking on an infinite regress and is therefore unsatisfactory. We are, therefore, left with (B3)—both of the remaining lines contain neither the whole part nor parts of the part.

What does (B3) entail? If both of the remaining lines do not contain any parts of the part, nor the whole part, we are forced to conclude that the original shared part between the two potential halves somehow ceases to exist when the potential parts are separated.[9] Now, if we then consider the two remaining lines as actually two lines in themselves, and yet consider each of them containing two potential halves within each of them, then upon the subsequent separation of these potential halves, these two lines will both lose another part of each. So it seems after every step in this process of halving lines, a part is lost. If this process

[9]It might here be objected that perhaps the part does not cease to exist; rather, it exists unshared and without magnitude. This further alternative will be examined later (in the examination of step 4).

continued to infinity and was completed, so as to correctly call the line 'infinitely divided', then an infinite number of parts would be lost. Thus, I believe it is possible to object to the assumption in premise (3) which holds that if a continuum is infinitely divided, then there will subsequently remain an infinite number of parts.[10]

The above objection implicitly assumes that premise (3) holds that a continuum can be infinitely divided into *all* of its parts. This is a necessary criterion for the objection to be viable. The premise as it stands, however, holds that a continuum can be infinitely divided into parts, not all of its parts. Nevertheless, the latter *may* necessarily follow from the former. This ultimately depends upon the meaning one attaches to the term 'divisibility', as I will argue below.

Premise (4)

Let us now suppose that the first three premises are wholly justified. Premise (4) has two claims: (i) it is impossible that there be an indivisible yet extended part, and (ii) it is impossible that a continuum be made up of an infinity of unextended parts. The assumption behind claim (i) is that all extended parts, as such, are divisible. This claim seems quite reasonable, at least when we consider extended entities such as lines. The alternate view is atomism. When we consider physical entities, atomism becomes more tenable, for perhaps it is impossible to divide a physical entity, even though it remains extended. However, I believe it is entirely impossible for a continuum's parts to have this property of indivisibility.

[10]The above objection may seem to be a variant of an argument by Adam of Wodeham (c. 1295–1358). In his *Tractatus de indivisibilibus*, he argues against the existence of indivisibles on grounds that one cannot actually divide a continuum into them. For after the first division, the resultant would be two different continua. Thus, what one would be continuing to divide would be something other than the original continuum. Although my objection to premise (3) might seem to have characteristics of this argument, it should remain distinct due to differences in purpose. Adam of Wodeham was interested in showing the implausibility of actually dividing a continuum, whereas I am interested in the ontology of a continuum given that it has already been infinitely divided.

Atomism originally emerged as an *a priori* solution to Zeno's paradoxes. It asserts that the basic units of continua are indivisibles with extent. The assumption behind atomism is that Zeno's paradoxes do not describe reality. In order to avoid the difficulties of Zeno's paradoxes, the atomists argue for the necessary existence of indivisibles with magnitude (Sorabji 338). For example, Sorabji writes, "Democritus . . . used the paradox of division everywhere as an argument for the existence of atomic magnitudes, that is, of magnitudes which, although having a positive size, are indivisible" (338). A version of Democritus's argument could take the following form:

1. If a magnitude is divided everywhere, the resultant parts will either be (i) indivisibles without magnitudes, or (ii) indivisibles with magnitudes.
2. If (i), then the original magnitude must have been composed only of entities with no extent, which is impossible.
3. If (ii), then the original magnitude was not fully divided everywhere, which contradicts (1).
4. Thus, no magnitude can be divided everywhere.
5. So there must be indivisible entities with magnitude, i.e., atoms.

We should here make a few necessary clarificatory distinctions. First, there is a distinction between types of divisibility. On the one hand, there is the concept of physical divisibility or actual divisibility. This refers to the extent to which one can actually "pull apart" segments from each other. For example, scientists may refer in this sense to the basic, constituent, and indivisible particles of matter. On the other hand, there is the concept of conceptual or potential divisibility. This concept refers to the extent to which one may, in theory, mentally divide magnitudes. In this mental sense, it seems plausible that even the basic building blocks of matter are possibly divisible into smaller parts.

Another distinction is between 'division everywhere' and 'infinite divisibility'. 'Division everywhere' is the concept of division used by Democritus in his arguments for atomism. For him, being divisible everywhere means simultaneous division, i.e., division into all constituent parts all at once. This form should be distinguished from the Aristotelian concept of 'infinite divisibility'. Aristotle's concept refers to a segment's ability to be divided into ever minute segments.

Let us return now to Democritus's argument for atomism. Democritus argues that division everywhere is not possible. Therefore, one must posit indivisibles with extent, i.e., atoms (this is the move from 4 to 5).[11] In spite of the fact that the Aristotelian position accords with the claim that division everywhere is impossible, it argues against the subsequent necessary postulation of atoms. That is, Aristotle would deny that the conclusion follows from premise (4). The reason is that when one applies the concept of infinite divisibility, there consequently remains an ever smaller magnitude. Democritus's argument "does not show any theoretical lower limit. . . . And to this extent Aristotle is right to insist . . . that you can still divide *anywhere*" (Sorabji 341). In other words, the postulation of atoms does not remain exempt from the problems caused by potential or conceptual divisibility. Therefore, I believe that (i) in premise (4) of the original argument is justified.

In the original argument, the second claim of premise (4) is based upon the assumption that no number of unextended parts (indivisibles), even if it is an infinite number, will ever give rise to a magnitude. I believe this claim also is quite plausible. The alternative to this view is indivisibilism. Indivisibilists hold that continua are composed of parts with no magnitude—indivisibles. Some of them, such as Robert Grosseteste and Henry of Harclay, hold that an infinite number of indivisibles make up continua, and others, such as Walter Chatton, hold that continua are composed of a finite number of indivisibles.

In *Physics* VI.1 (231a 21–231b 8), Aristotle argues against indivisibilism by proposing that any number of indivisibles could not be continuous or in contact with each other.[12] For Aristotle, two things are continuous only when their extremities are one.[13] Since indivisibles have no parts,

[11]This of course depends upon whether one accepts the prior premises. It seems to me that of these, only (2) is arguable. Whether (2) is an impossibility or not will be further examined when we examine arguments for indivisibilism.

[12]He does, however, argue that indivisibles could be in succession to one another, but not in such a manner to form a continuum.

[13]If continuity is defined as such, i.e., extremities are one, it seems unclear how there originally existed two distinct things. But even if we discard his argument against the continuity of indivisibles, Aristotle's "touching argument" remains pressing.

by definition, they can have no extremities. Thus, no two indivisibles could have extremities which are one. Moreover, Aristotle suggests the only manner that indivisibles could be in contact with one another would be in such a manner so as to produce no extent. Since indivisibles have no parts, the only way they could touch each other would be whole to whole (as opposed to part to part). If, however, they touched each other whole to whole, they could not produce any magnitude.[14]

The medieval indivisibilists tried to meet these powerful arguments in various ways. Harclay, for example, attempted to refute the idea that if indivisibles were in contact with one another, they would not produce any quantum. In William of Alnwick's *Determinatio II*, Harclay argues that although indivisibles cannot produce an increase in size if touching whole to whole "in the same local position," if they touched whole to whole, "in distinct positions immediately next to one another," a quantum would result (321). However, it seems to me in order to produce an increase in size, not only do things need to be immediately next to each other, but also both things must already have magnitude. This objection strengthens Aristotle's position for it suggests that indivisibles cannot touch each other in any other respect than whole to whole, superimposed upon one another. Because indivisibles entirely lack extent, to be touching whole to whole in distinct positions immediate to one another is just touching whole to whole in the same local position (superimposed upon one another). Despite the failure of Harclay's endeavors, many other philosophers have attempted to form plausible indivisibilist arguments, probably to explain angelic motion or the existence of unequal infinities (Murdoch 576–77).

Perhaps my prior objection to premise (3) might be better constructed as an indivisibilist argument itself, an argument which would attempt to show, as did Grosseteste and Harclay, that an infinite number of indivisibles make up continua. It was assumed throughout that argument that parts of lines must have extent. But if assumed, it becomes unclear how, if one of these parts is lost in the process of division, i.e., the conclusion from (B3), the summation of remaining lines could be equal to that of the original line. Thus, I should have also

[14]At least any magnitude greater than that of one indivisible—which is nonetheless defined as extensionless.

considered another alternative to (B3), i.e., that the parts in question may be parts without magnitude—indivisibles.

This indivisibilist picture of the remaining parts of infinite division would be significantly different from the conclusion drawn from (B3) in my above objection to premise (3). The main difference for the indivisibilist is that at each step in the process of infinitely halving a line, the part that originally was shared has no extent or magnitude (this is the further alternative to B3). Thus, when the two potential halves are actually separated, they do not lose any of their total magnitude, yet the shared part ceases to be shared nonetheless. So, what remains after the first division is two lines and an unshared part without magnitude. As this process is repeated, the magnitudes of the remaining lines get smaller and smaller, and the number of remaining unshared parts without magnitude gets larger and larger. It would seem to follow that ultimately what results from this process would be an infinite number of unshared parts without magnitude.

But regardless of which form of indivisibilism one might hold, there ultimately remains the significant problem of how entities with no magnitude can possibly compose an entity with magnitude. A mathematical approach may best explicate the problem. Zero multiplied by any finite number, or infinity, will necessarily produce zero. Hence, something with zero magnitude, even if multiplied an infinite number of times, will not yield something with nonzero magnitude. Therefore, in spite of the above argument, there are further reasons to suppose indivisibilism in any form to be implausible. We should, therefore, conclude that both claims (i) and (ii) in (4) are justified assumptions.

Premises (5) and (6)

Premises (5) and (6) are mere logical consequences from the first four premises in the argument. Thus, if we feel that these prior premises are justified, by modus tollens, we must necessarily hold that it is logically impossible to suppose a continuum to have been infinitely divided. Hence, a continuum cannot be infinitely divided.

Conclusion (7)

If the argument intended to prove that a continuum cannot be infinitely divided, then it would be a valid argument. However, this is not the case. The argument originally set out to prove that a continuum cannot be infinitely *divisible*. The move from premise (6) to the conclusion (7) reflects the idea that the latter claim is somehow deducible from the former. But it is altogether unclear how the impossibility of an infinitely divided continuum entails that a continuum cannot be infinitely divisible.

On Aristotle's account, the infinite divisibility of a continuum implies a continuing and never-ending process of division. That is to say, an infinite process of division *can be* occurring, not that it has already been or could be completed (making the continuum infinitely divided). However, if divisibility is defined in this way, then the entire argument fails because this conclusion does not follow from premise (6). To think it does would be to conflate the process of dividing a continuum *ad inifinitum* with the completion of that process. While it would necessarily be the case that if a continuum were infinitely divided, it would also be infinitely divisible, the converse nonetheless remains dubious.

The reason that Aristotle holds this view of infinite divisibility and not an alternate view, like 'division everywhere', can be understood by noting his views on the ontology of the infinite found in *Physics* III. It is here that Aristotle denies the existence of actual infinities and argues the infinite can exist only potentially. He describes infinity in terms of approaching limits or finitude.[15] Thus, the infinite is "not what has nothing outside it . . . but what always has something outside it" (*Physics* III.6 207a1–2). Seen in this light, the infinite can never *actually* be more than finite, yet potentially it ever continues to extend. Thus, infinity can only potentially exist.[16]

Infinite division, likewise, can only potentially be completed. Only a finite number of divisions could actually occur within a given

[15]This is the position that Sorabji takes on the matter. He writes, "Infinity is an *extendible finitude*" (210).

[16]Aristotle distinguishes between two forms of potentials: (1) potentials which can become fully actualized, e.g., a potential statue, and (2) potentials which are always coming into being, e.g., days (*Physics* III.6 206a20–206b). Aristotle's concept of infinity falls under (2). Thus, Aristotle writes, "Generally the infinite has this mode of existence: one thing is always being taken after another, and each thing that is taken is always finite, but always different" (*Physics* III.6 206a26–30).

magnitude, yet there will always remain the potential of more actual divisions. Concerning infinite divisibility, Aristotle writes, "the sum of the parts taken will not exceed every determinate magnitude, just as in the direction of division every determinate magnitude is surpassed in smallness and there will be a smaller part" (*Physics* III.6 206b17–19). In this manner, we can see that Aristotle's notion of infinite division is similar to his conception of the infinite defined as approaching a limit.

In this essay, I have attempted to show that the initial argument presented above fails. Firstly, there are reasons for denying either the assumption in premise (3) or part (ii) of premise (4). If a continuum has been infinitely divided, then there is the further possibility that actually there would exist either no remaining parts (contra premise 3) or an infinite number of indivisibles (contra part [ii] of premise 4). Even if one does not accept this objection to (3) or to part (ii) of (4), the argument clearly fails elsewhere. In spite of the fact that the argument from (1) to (6) is valid, the move from premise (6) to the conclusion (7) is unwarranted. The impossibility of the infinite divisibility of a continuum is simply not deducible from the impossibility of an infinite division of a continuum. Hence, by assessing the original argument and the many perspectives from which people have traditionally looked at the problems concerning the infinite divisibility of continua, I ultimately endorse an Aristotelian form of divisibilism.

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